General Identifiability with Arbitrary Surrogate Experiments

Sanghack Lee
with Juan Correa and Elias Bareinboim

Columbia University

AAAI 2020
(presented at UAI 2019)

Overview

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- Causal Effect **Identifiability** concerns about precisely determining the effect of intervention given information (e.g., causal assumptions and an observational data).
- **General Identifiability** considers identifying a causal effect given an arbitrary combination of observational and experimental data .
- We provided a graphical **necessary and sufficient condition** under which a causal effect of interest can be estimable. We devised a **sound and complete algorithm** which outputs a formula for the causal effect made with probabilities obtained from available data.

Understanding Data

unknown

real-world

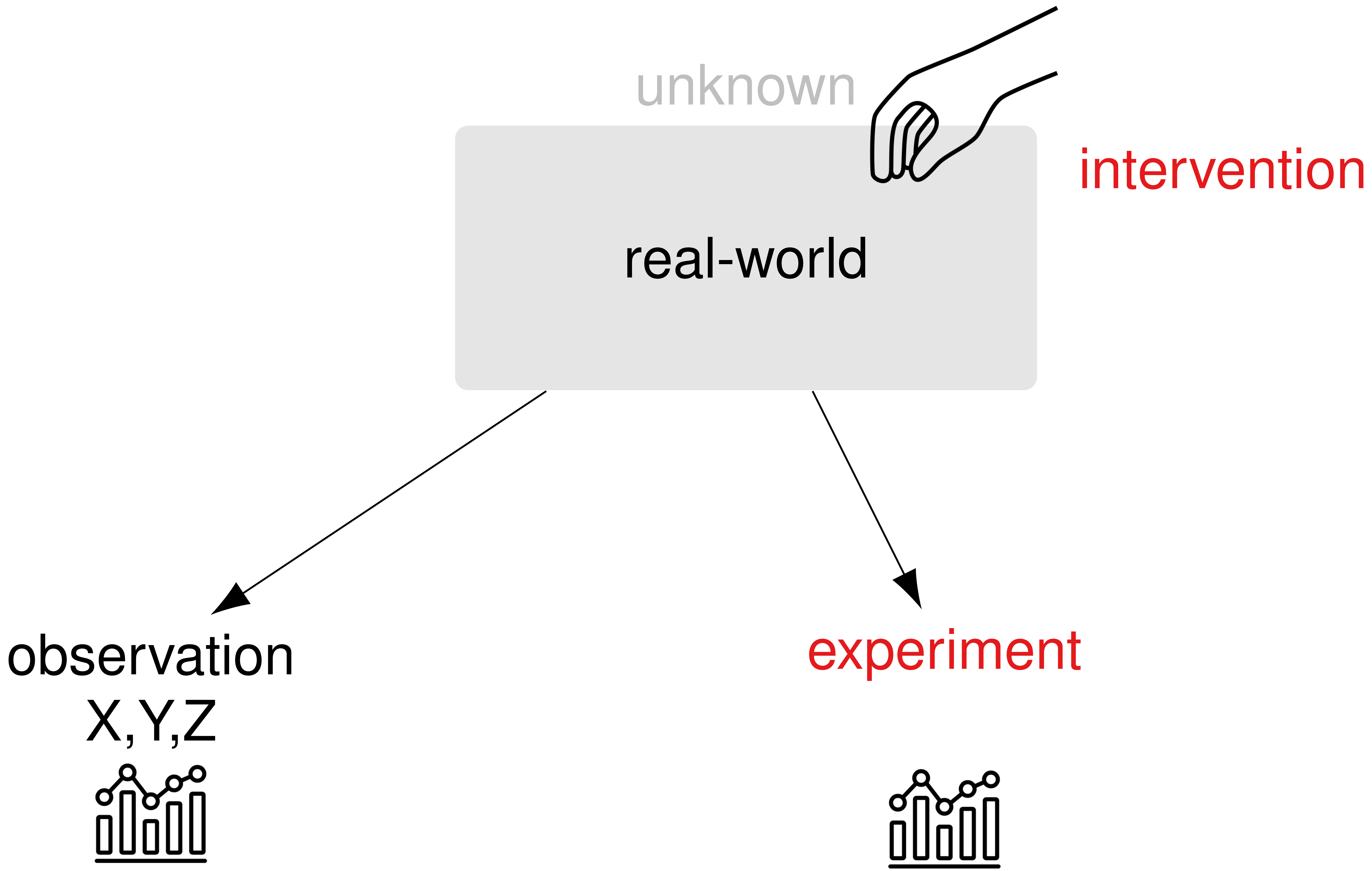
Understanding Data

unknown

real-world

observation
X,Y,Z

Understanding Data



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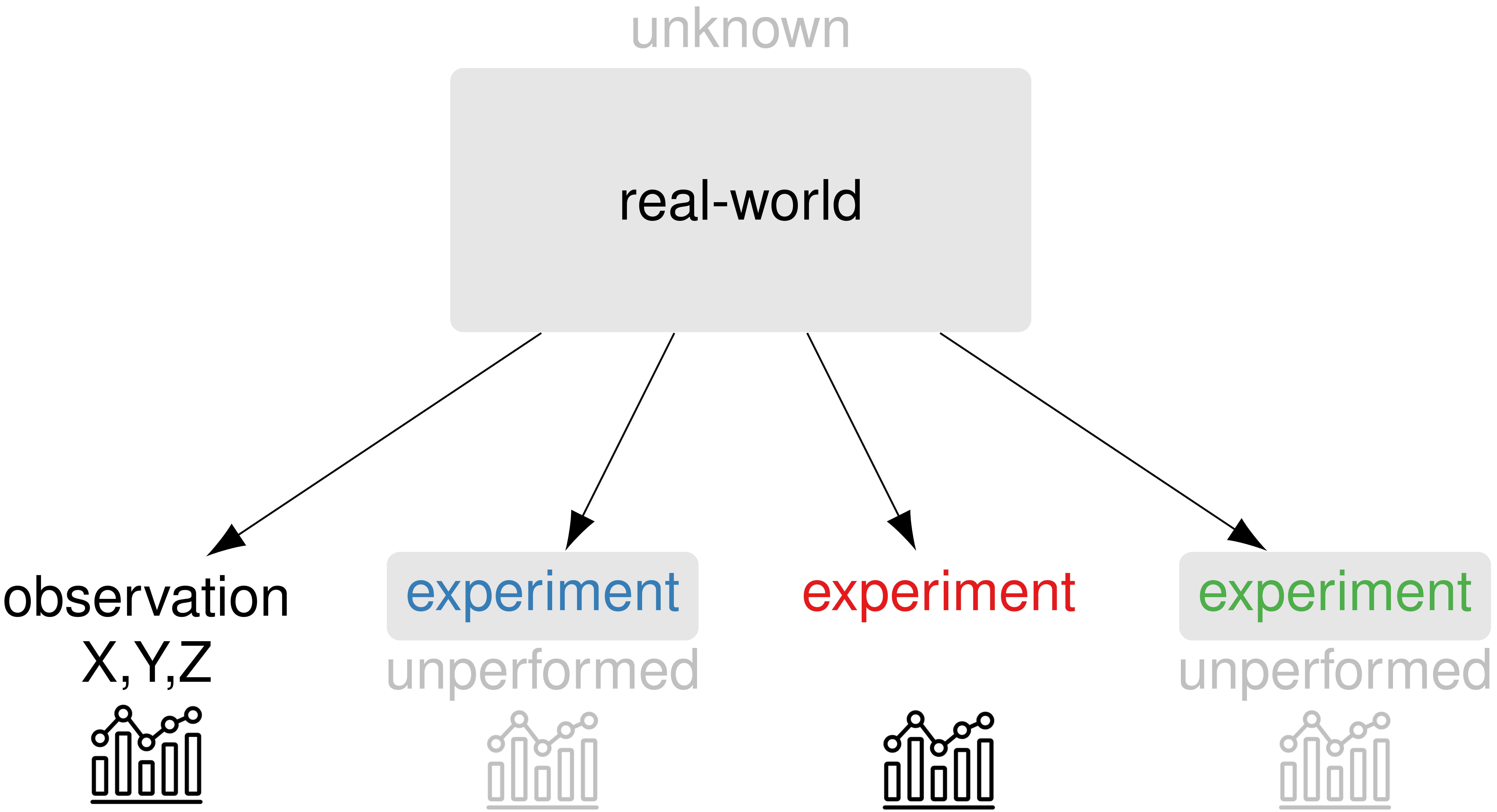

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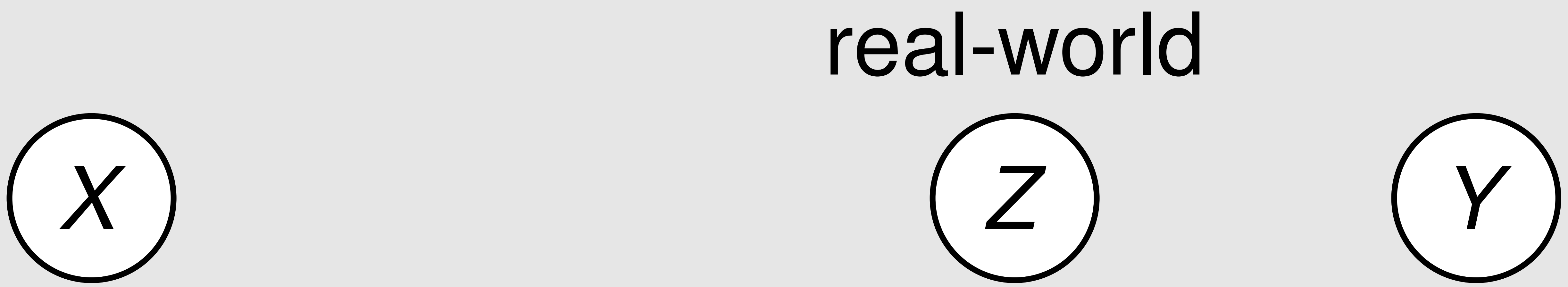

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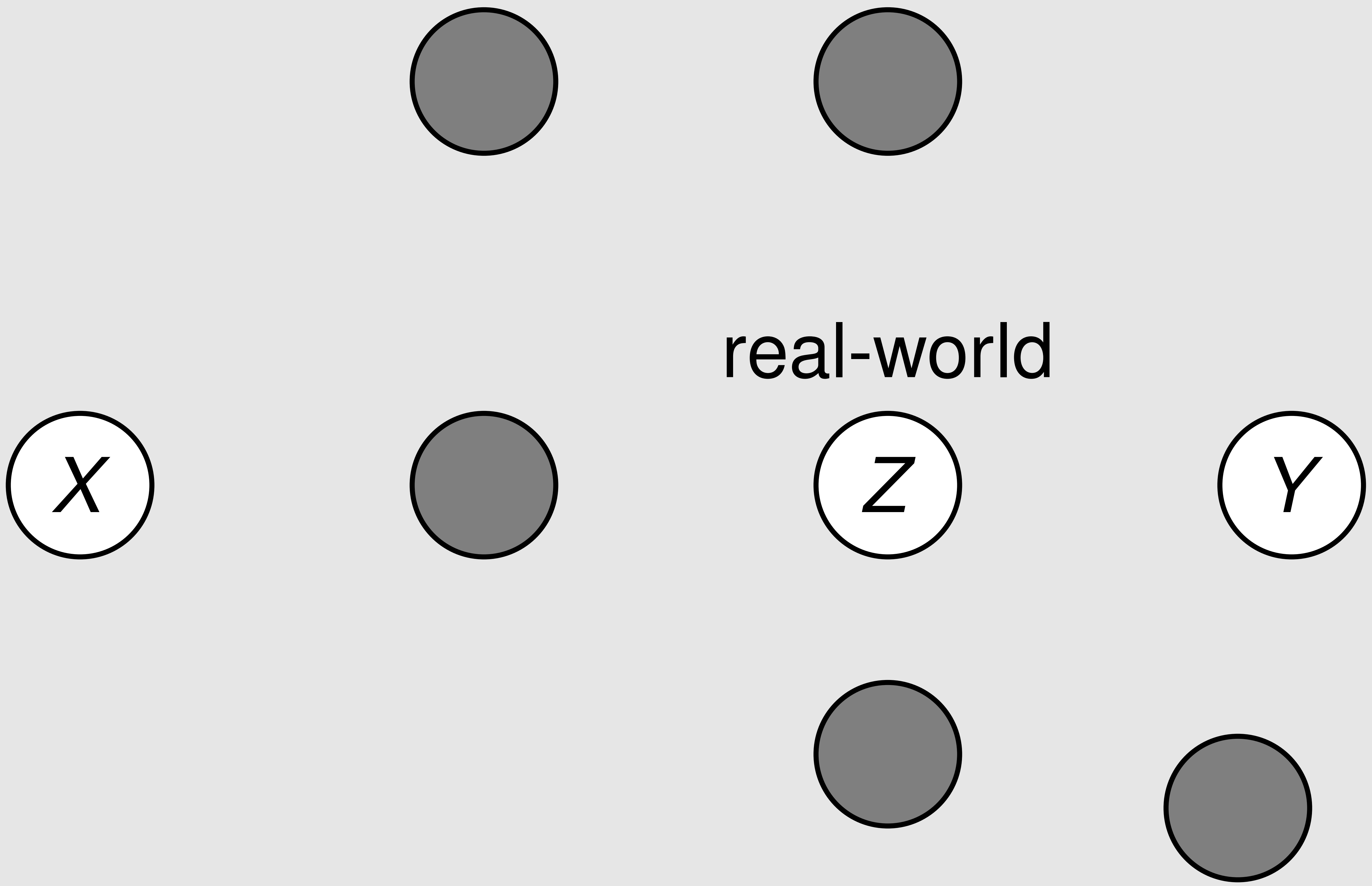

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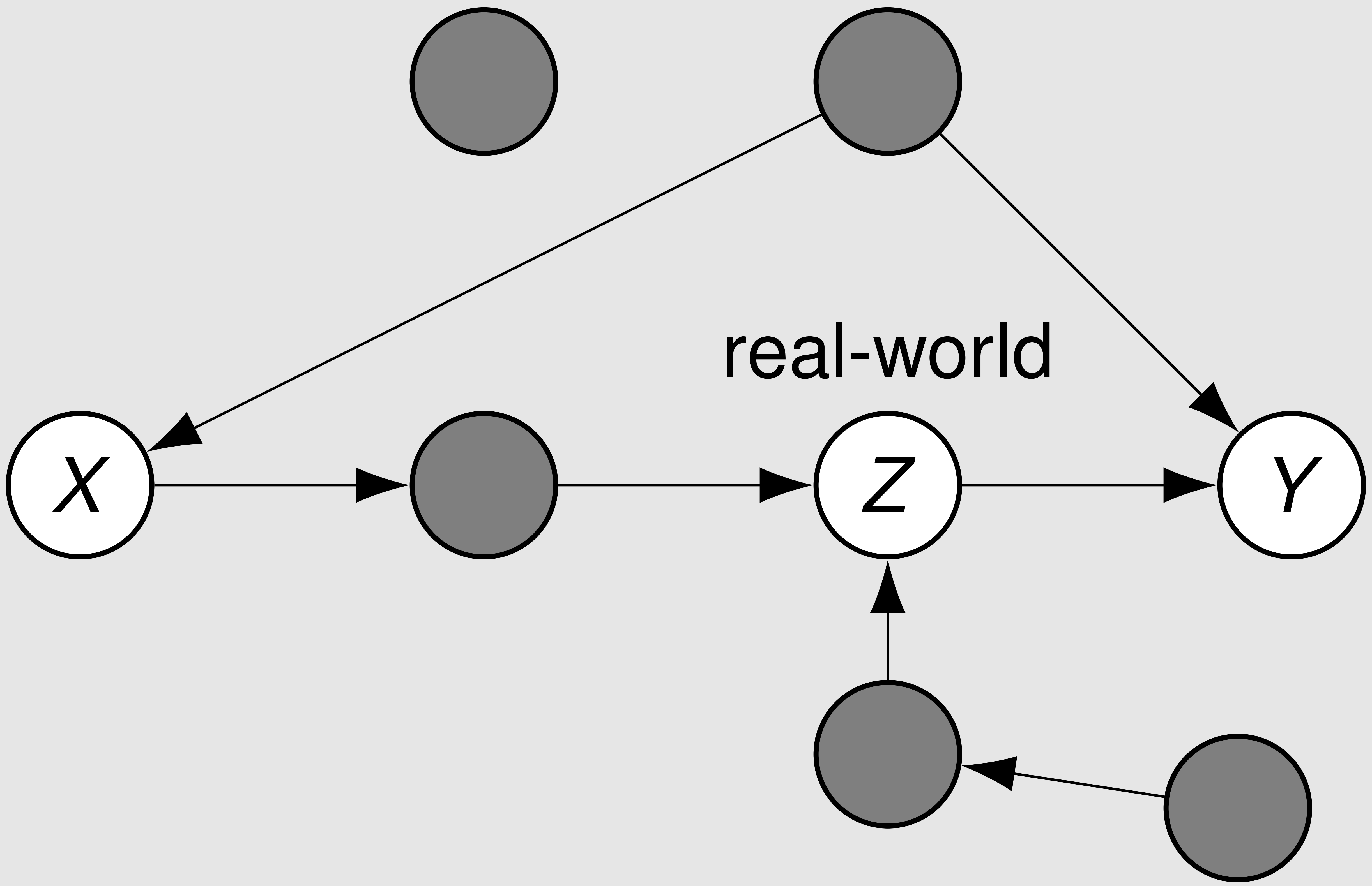

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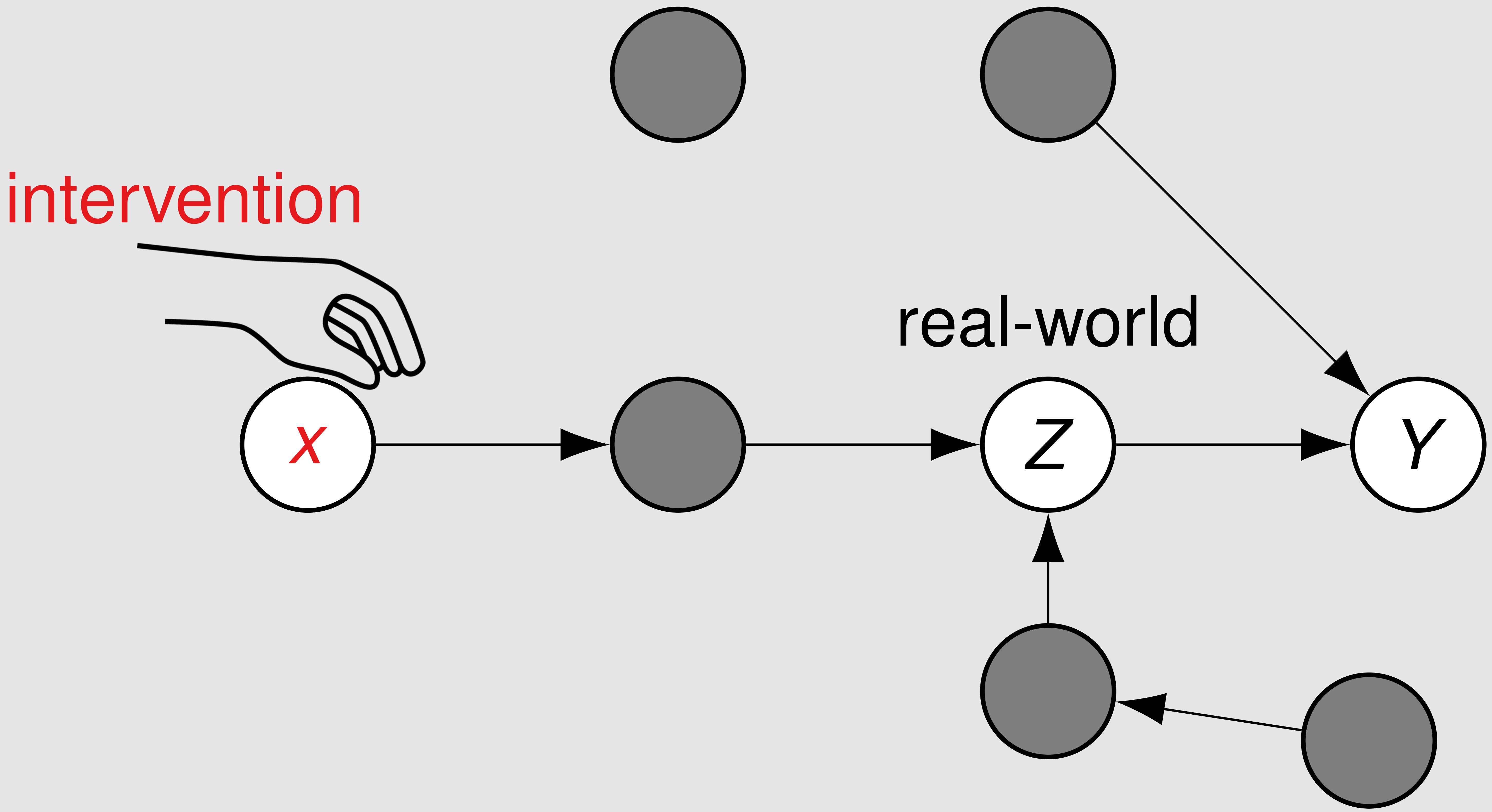

Causal Framework

SCM provides an abstraction of causality in the real-world.

Definition (Structural Causal Model (Pearl))

SCM $\mathcal{M}$ is a 4-tuple $\langle \mathbf{U}, \mathbf{V}, \mathbf{F}, P(\mathbf{U}) \rangle$

- $\mathbf{U} = \{U_1, \ldots, U_m\}$ are **exogenous** variables;
- $\mathbf{V} = \{V_1, \ldots, V_n\}$ are **endogenous** variables;
- $\mathbf{F} = \{f_1, \ldots, f_n\}$ are **functions** determining $\mathbf{V}$,

$$v_i \leftarrow f_i(\mathbf{pa}^i, \mathbf{u}^i)$$

  where $\mathbf{PA}^i \subseteq \mathbf{V} \setminus \{V_i\}$, $\mathbf{U}^i \subseteq \mathbf{U}$; and
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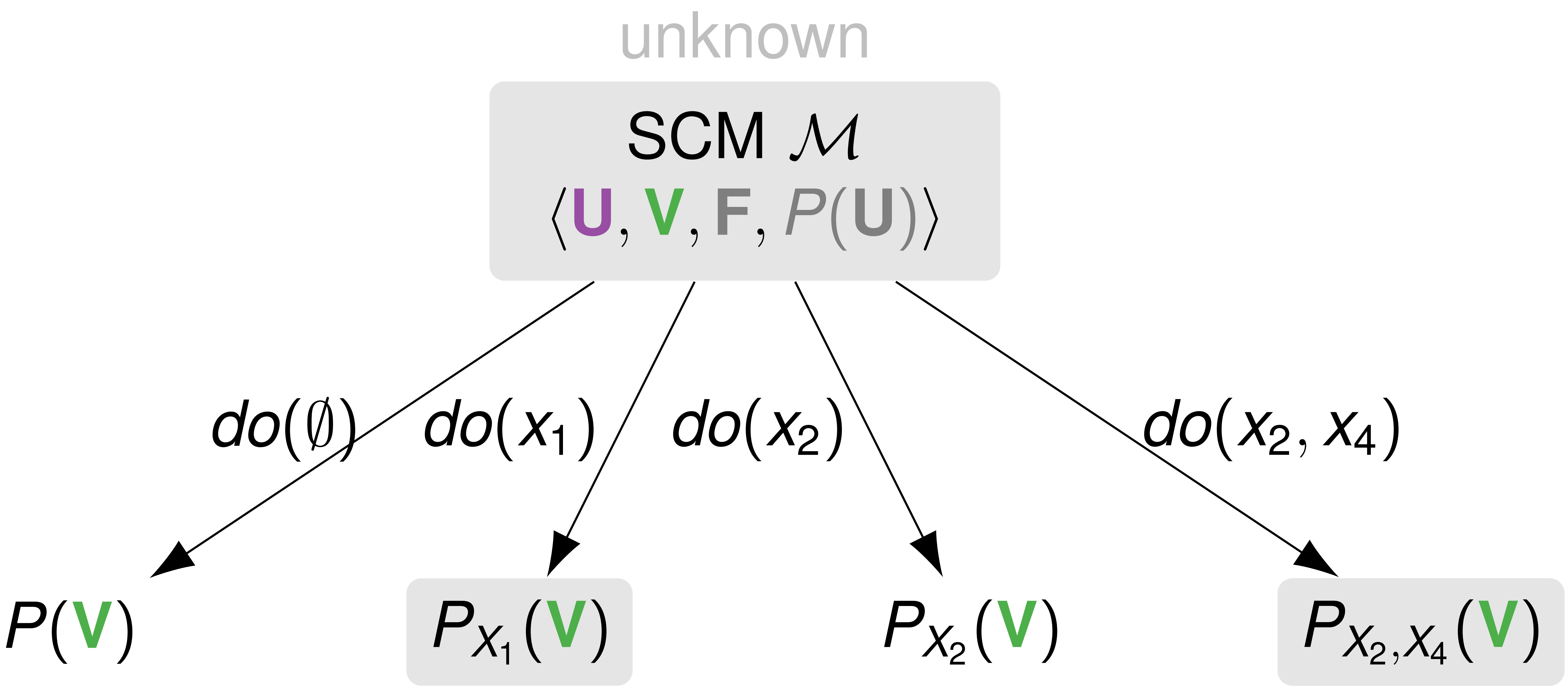

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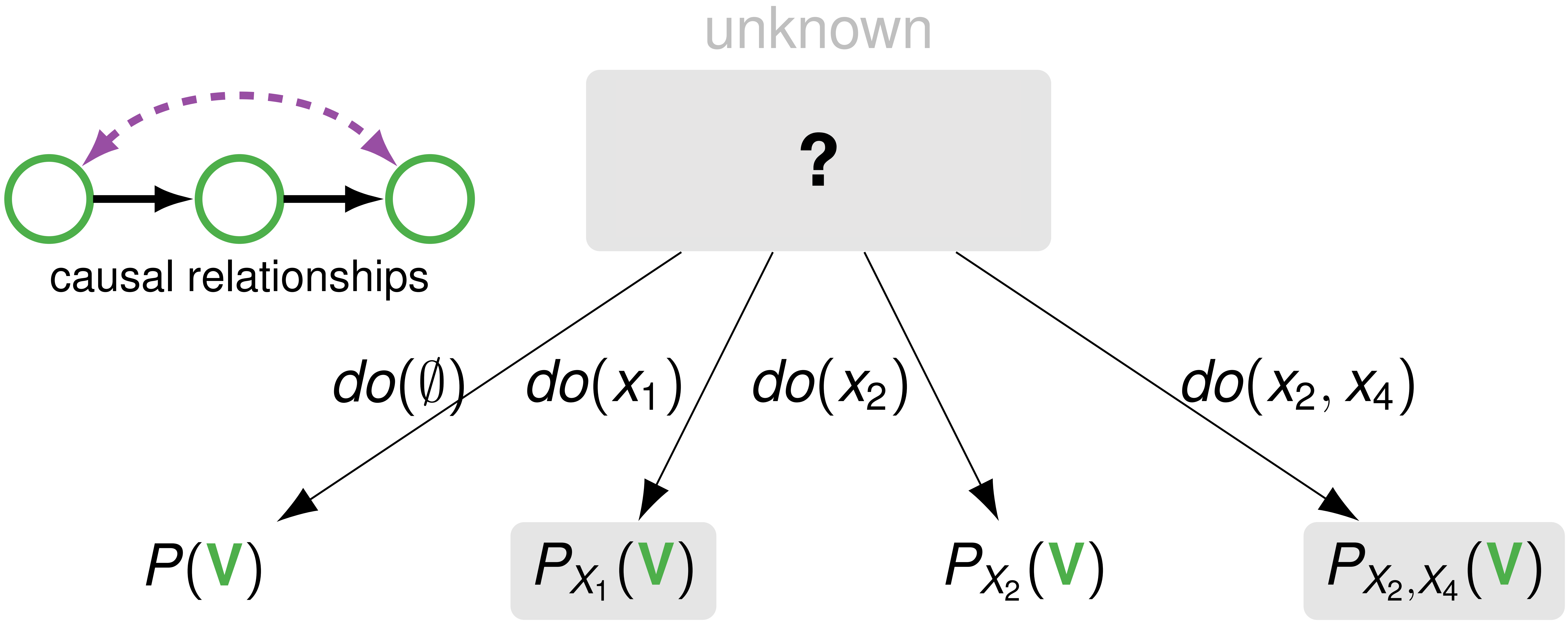

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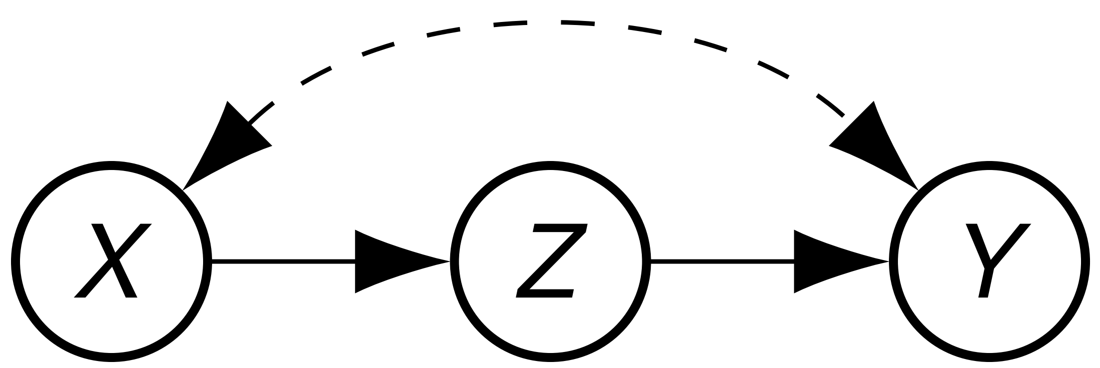
1. Query Q

   $P_{\mathbf{x}}(\mathbf{y}) = P(\mathbf{y}|do(\mathbf{x}))$

2. Causal Diagram $\mathcal{G}$



3. Data

   $P(\mathbf{V})$

a non-parametric assumption: no assumption on P

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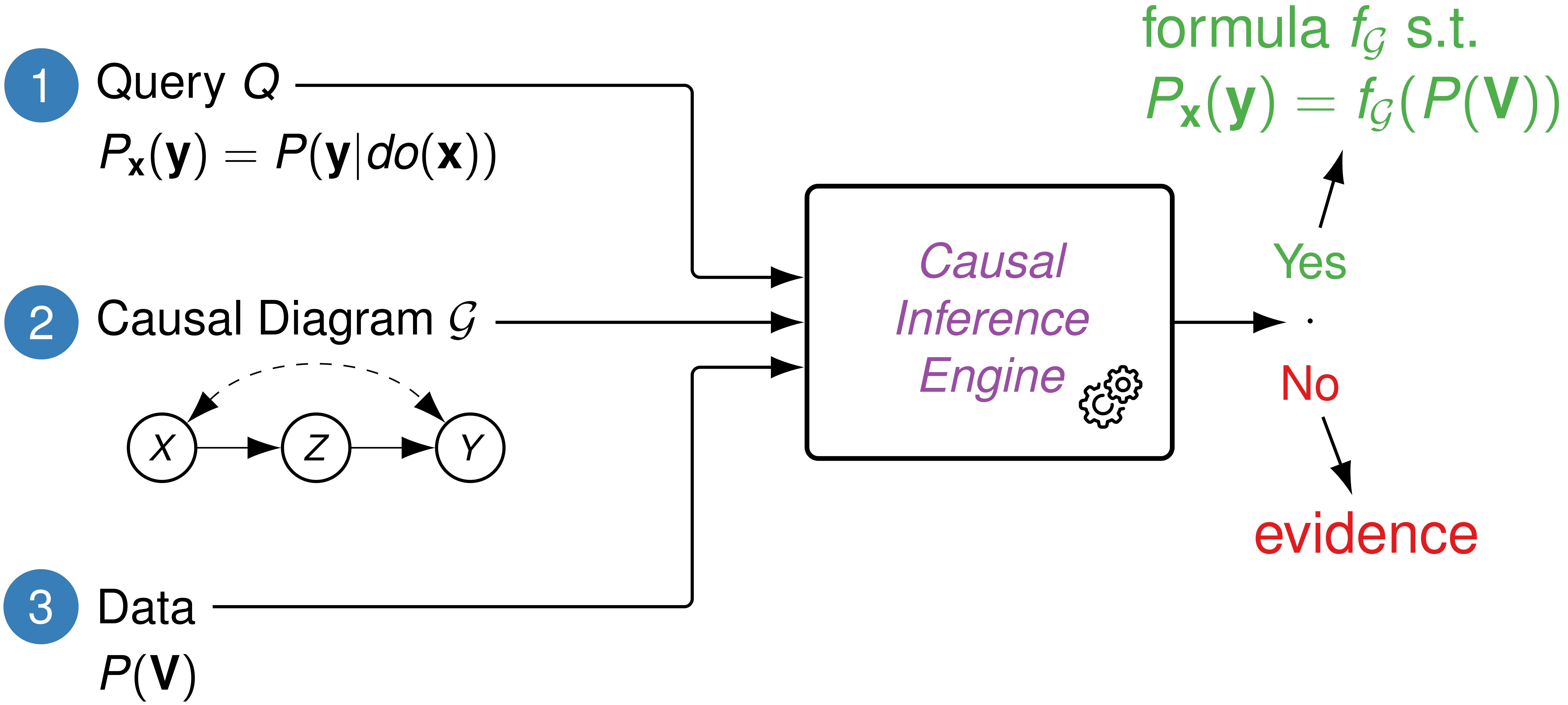

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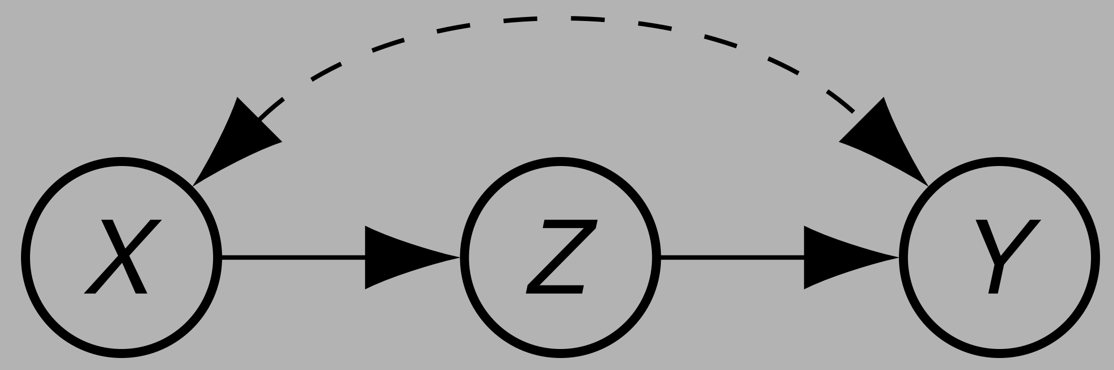

3 Data

arbitrary combinations?

Causal Inference Engine

Yes

formula $f_{\mathcal{G}}$ s.t.

$P_{\mathbf{x}}(\mathbf{y}) = f_{\mathcal{G}}(P(\mathbf{V}))$

No

evidence

a non-parametric assumption: no assumption on P

g-identifiability

Definition (g-Identifiability)

Let $\mathbb{Z} = \{\mathbf{Z}_i\}_{i=1}^{m}$ be a collection of sets of variables.

$P_{\mathbf{x}}(\mathbf{y})$ is said to be **g-identifiable** from $\mathbb{P}$ in $\mathcal{G}$, if $P_{\mathbf{x}}(\mathbf{y})$ is **uniquely computable** from distributions

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Causal identifiability has been been studied in the literature:

ID An observational distribution [TP'02, SP'06, HV'06]:

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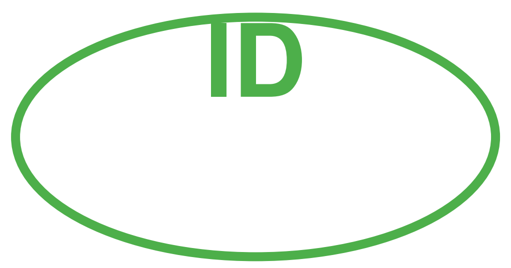

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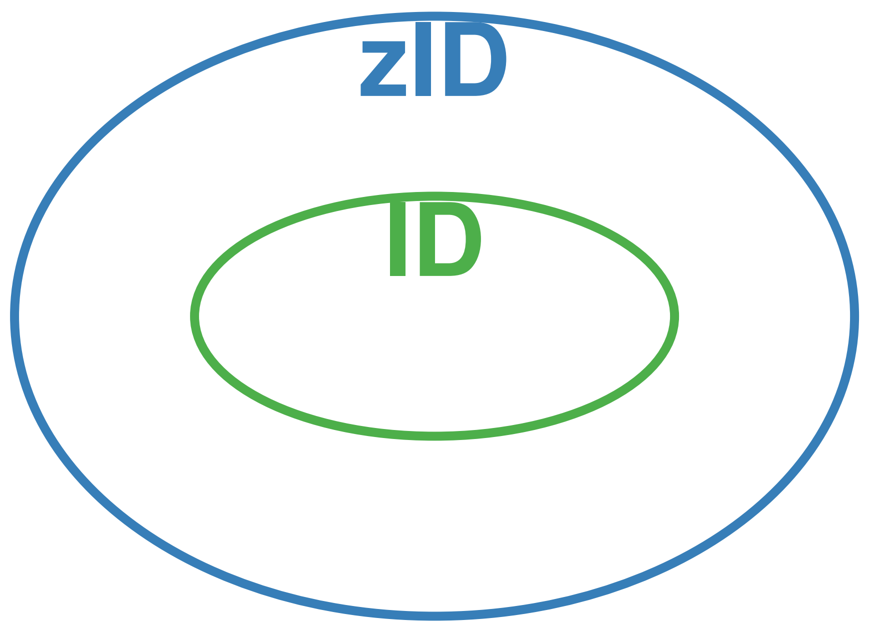

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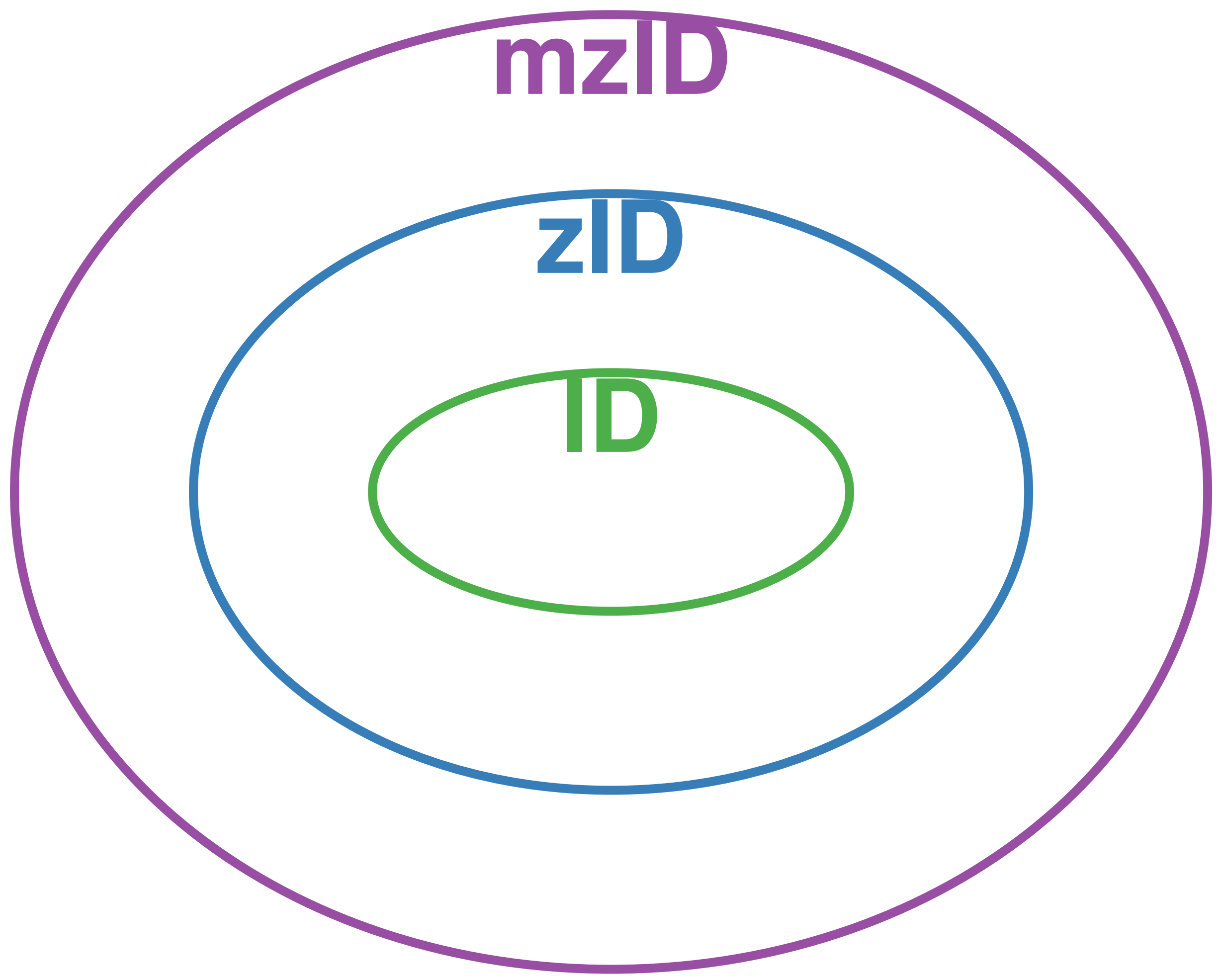

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gID A collection of arbitrary experiments [LCB'19]:

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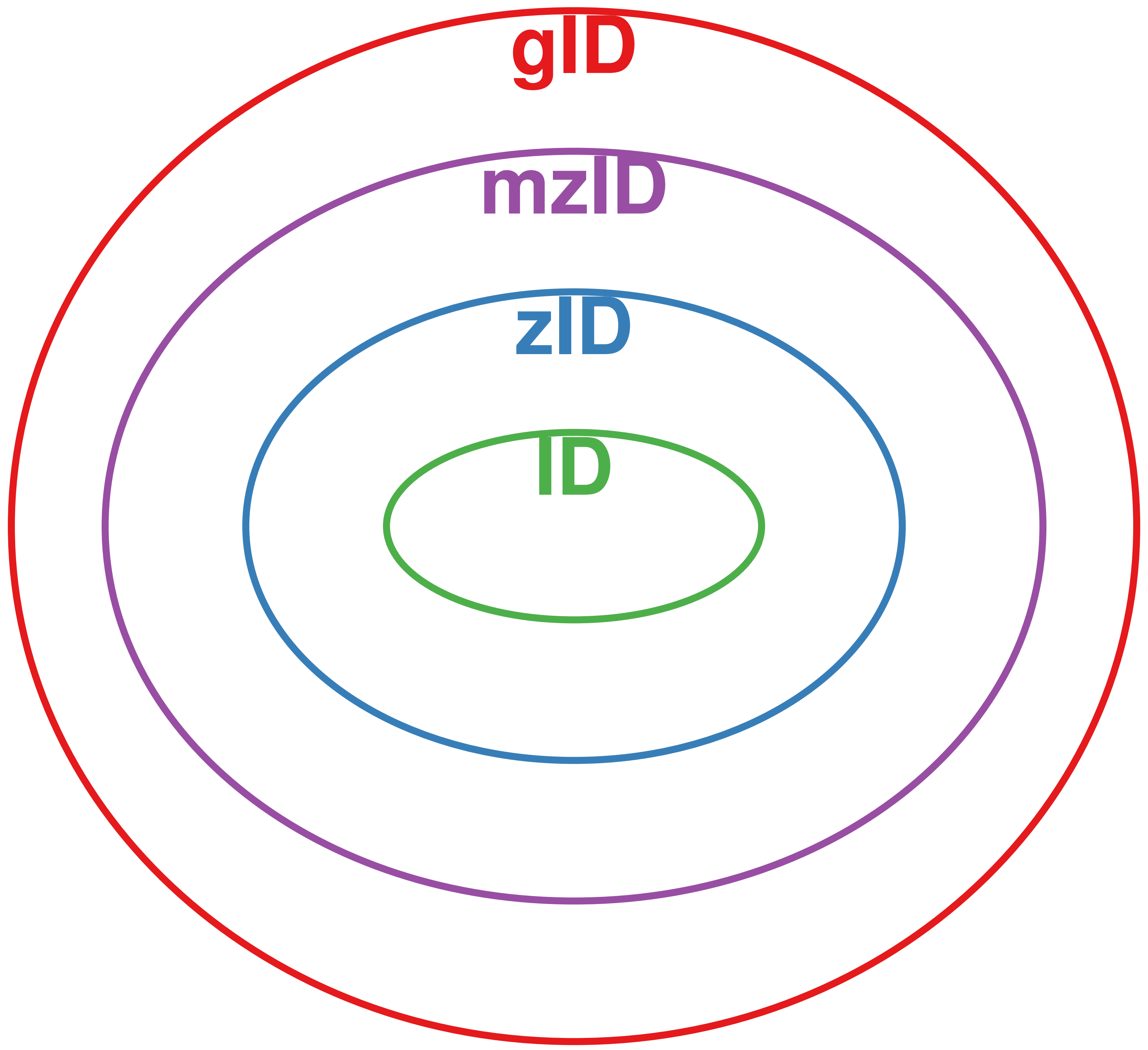
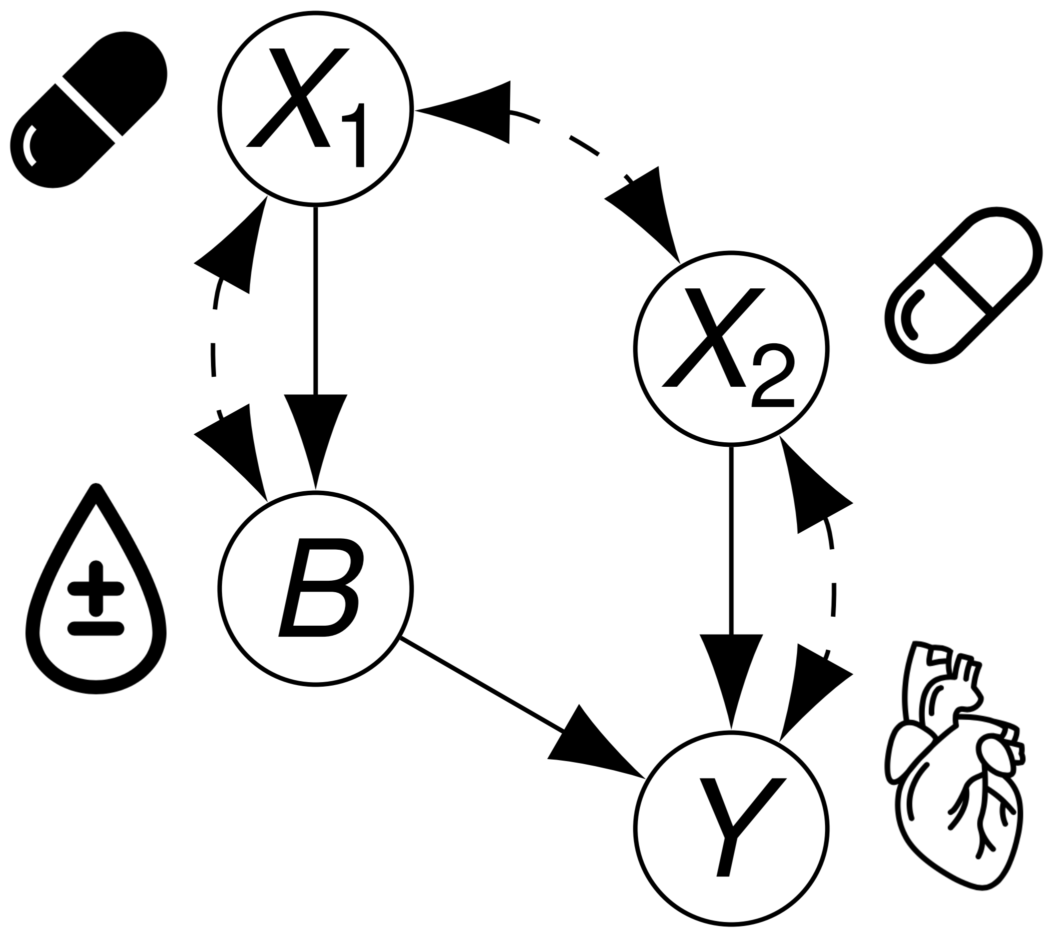


Example: Drug-Drug Interactions



(a)

Y *cardiovascular disease*; B *blood pressure*; X_1 taking an *antihypertensive drug*; and X_2 the use of an *anti-diabetic drug*.

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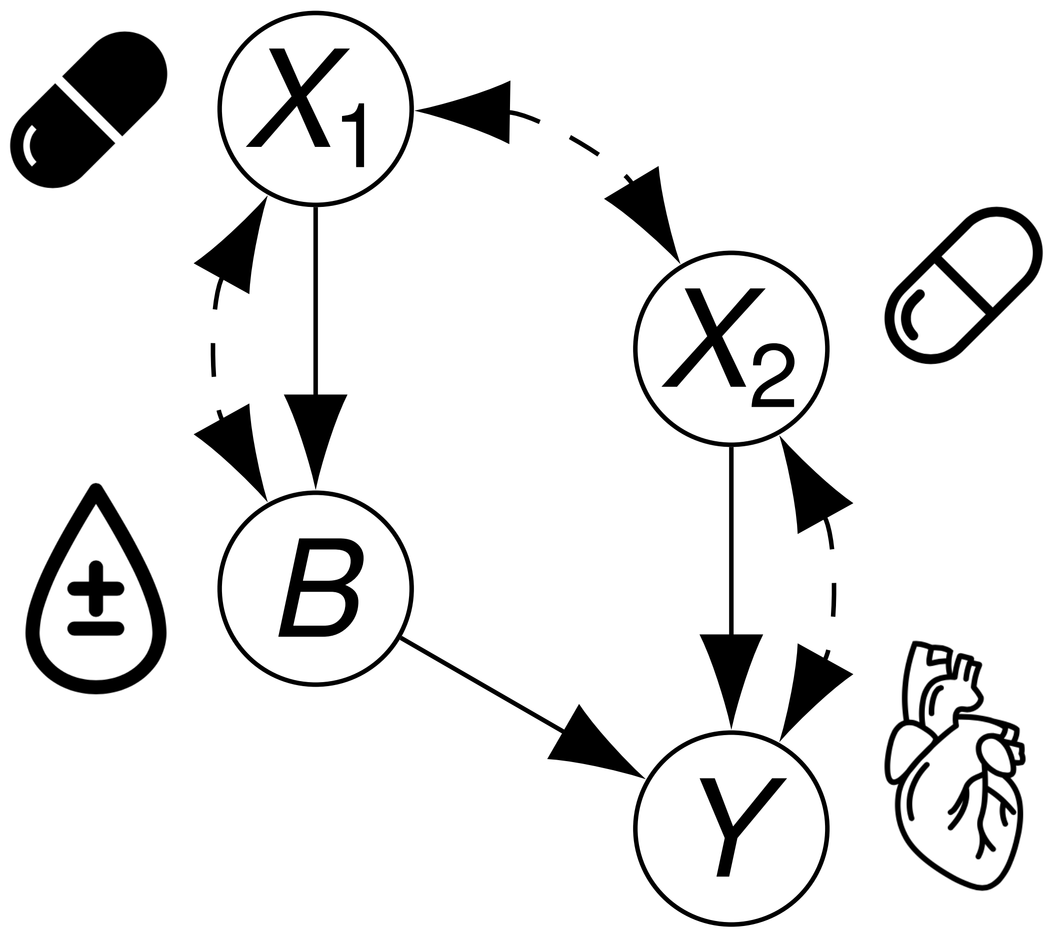

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$$P_{x_1,x_2}(y) \Leftarrow \{P_{x_1}(\mathbf{V}), P_{x_2}(\mathbf{V})\}?$$

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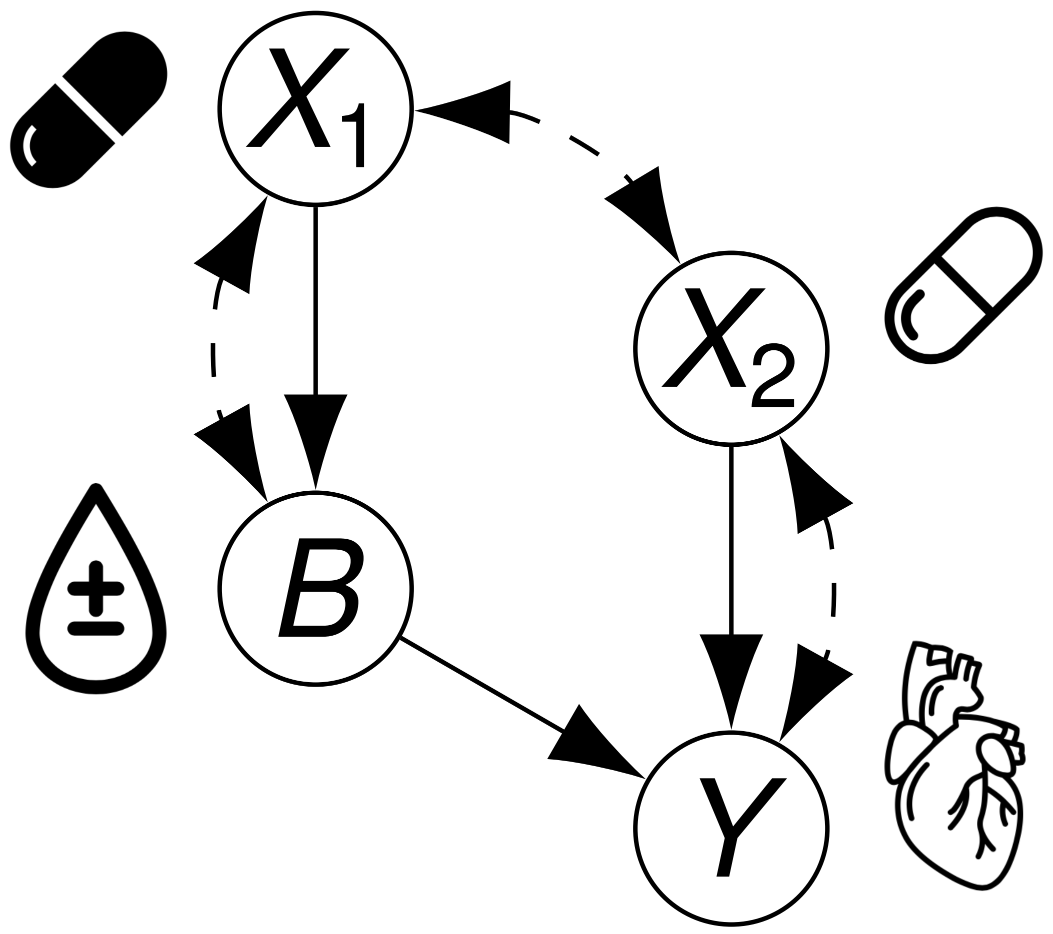

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$$P_{x_1,x_2}(y) = \sum_b P_{x_2}(y|b)P_{x_1}(b)$$

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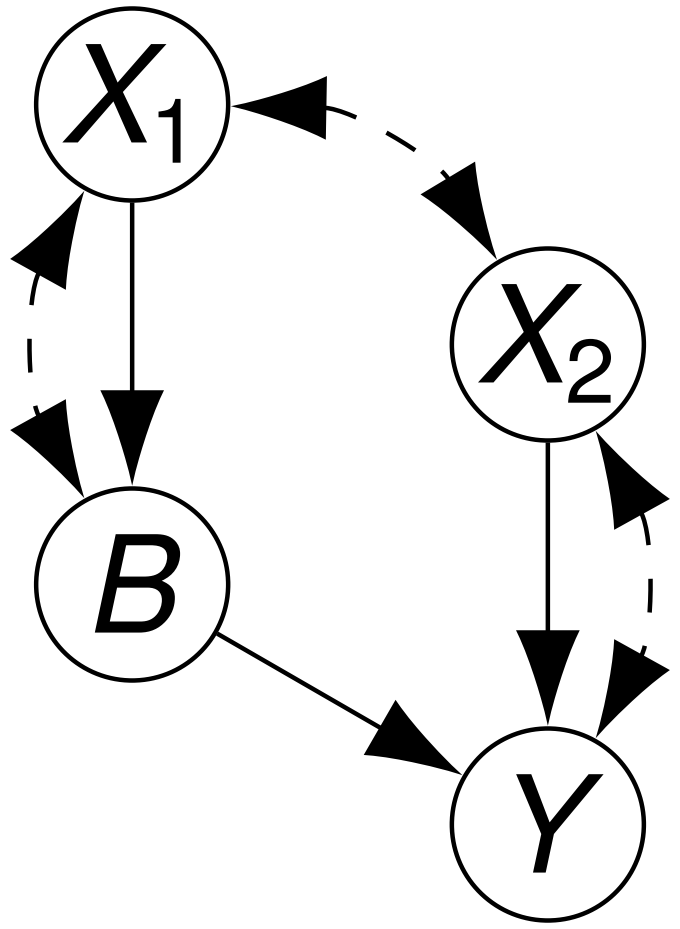

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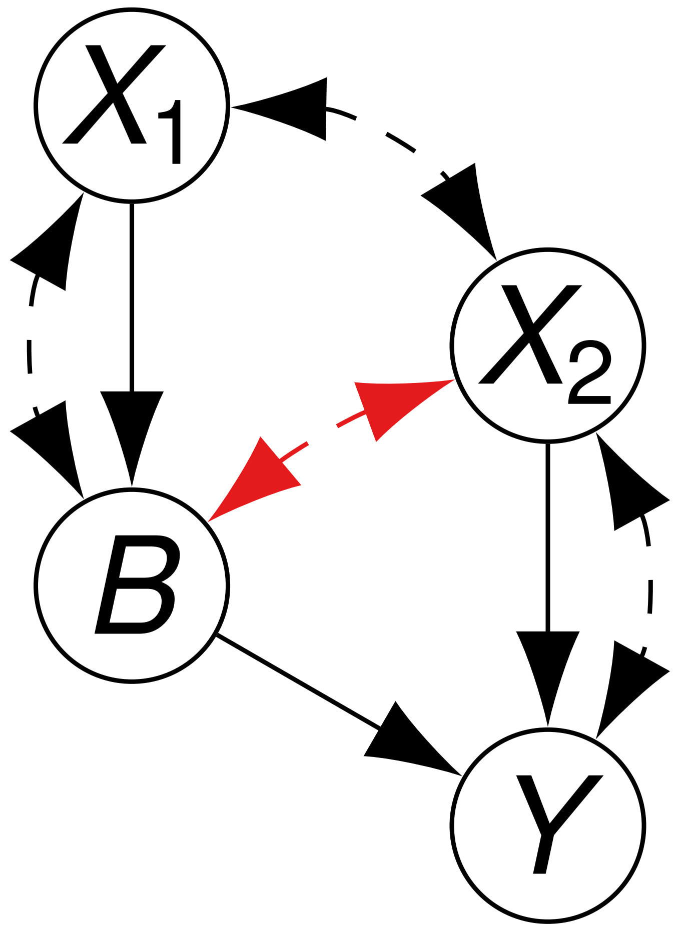

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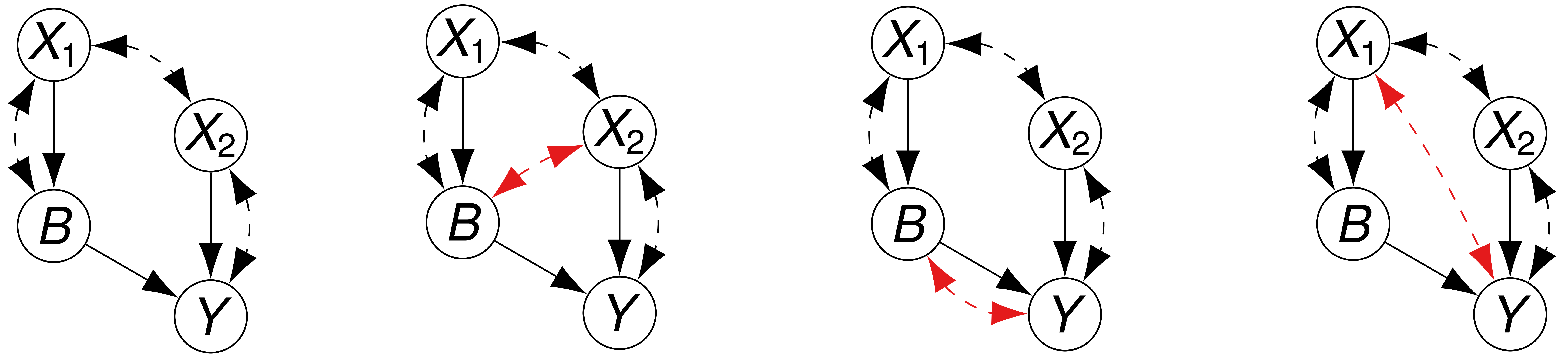

(a) ✓ (b) ✓ (c) ✗ (d) ✗

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g-identifiability
– a sound algorithm

Algorithm for gID

We developed a two-phase algorithm w/ probability axioms & do-calculus:

1. A given query is modified, and factorized into subqueries;
2. **Each** subquery is identified by **one** of the available distributions.

* It FAILs **only if** there exists a subquery that cannot be identified by any of the available distributions.

$Q \quad P_{\mathbf{x}}(\mathbf{y})$

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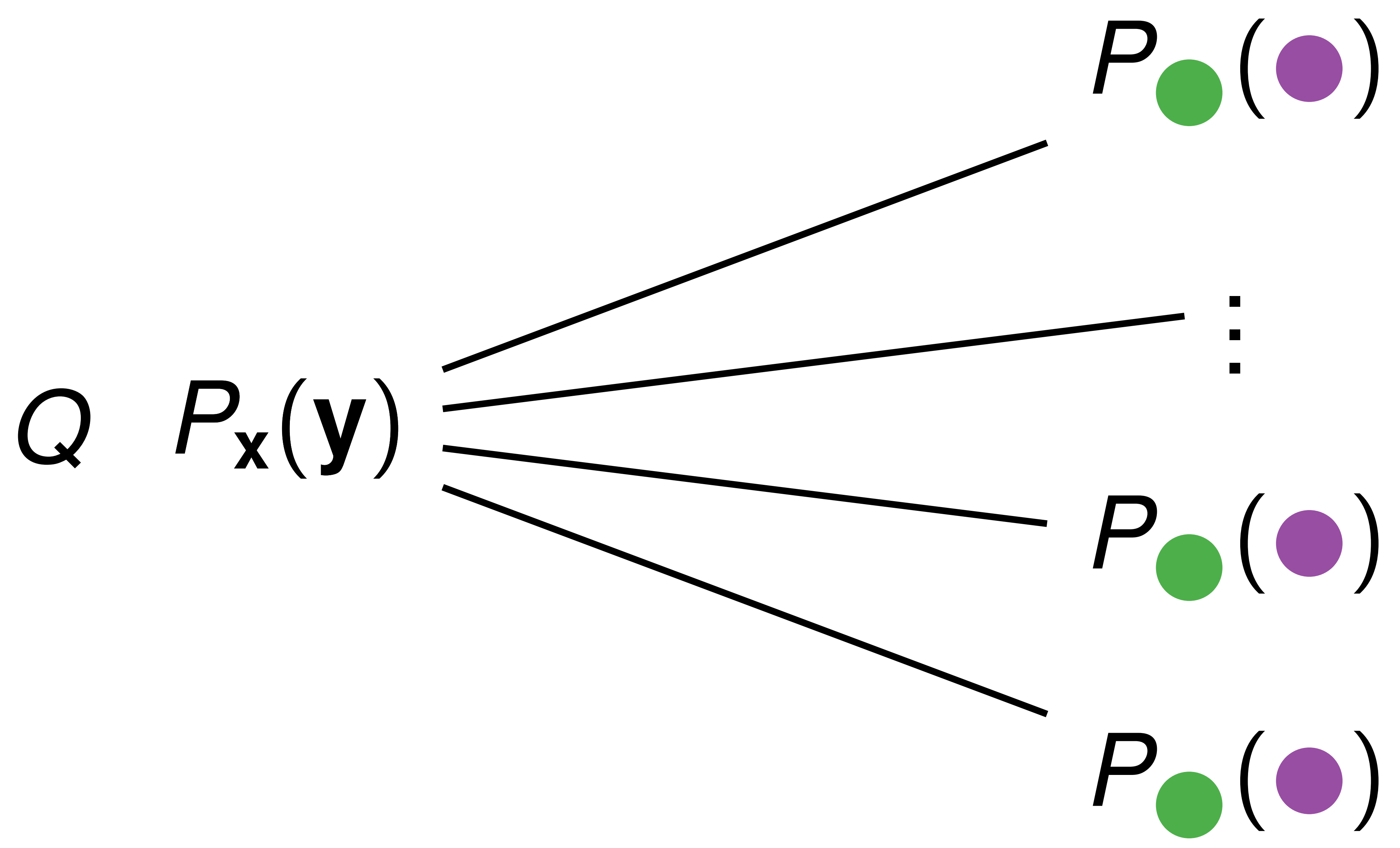

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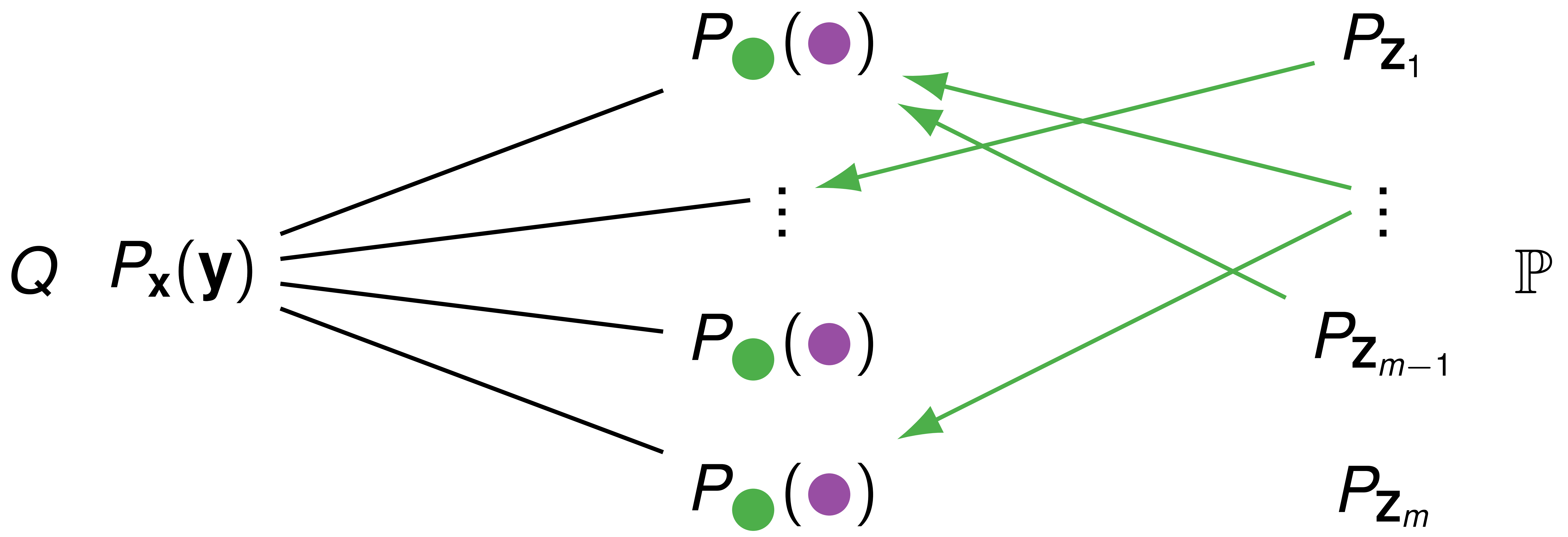

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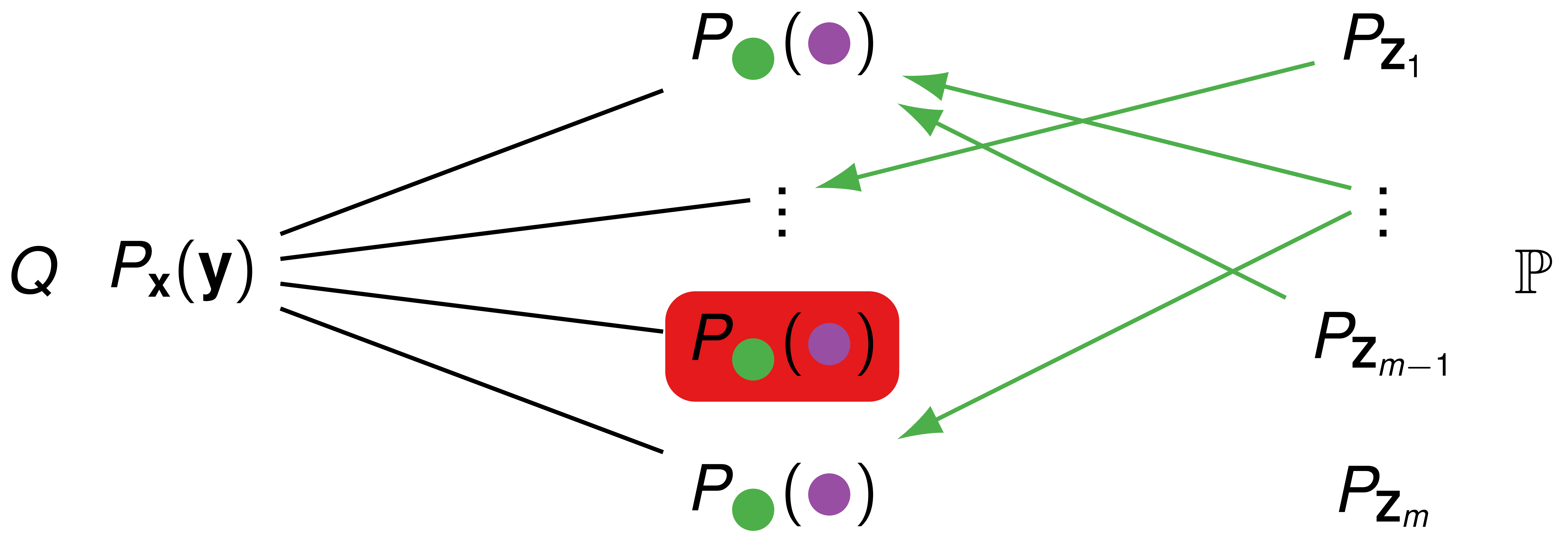
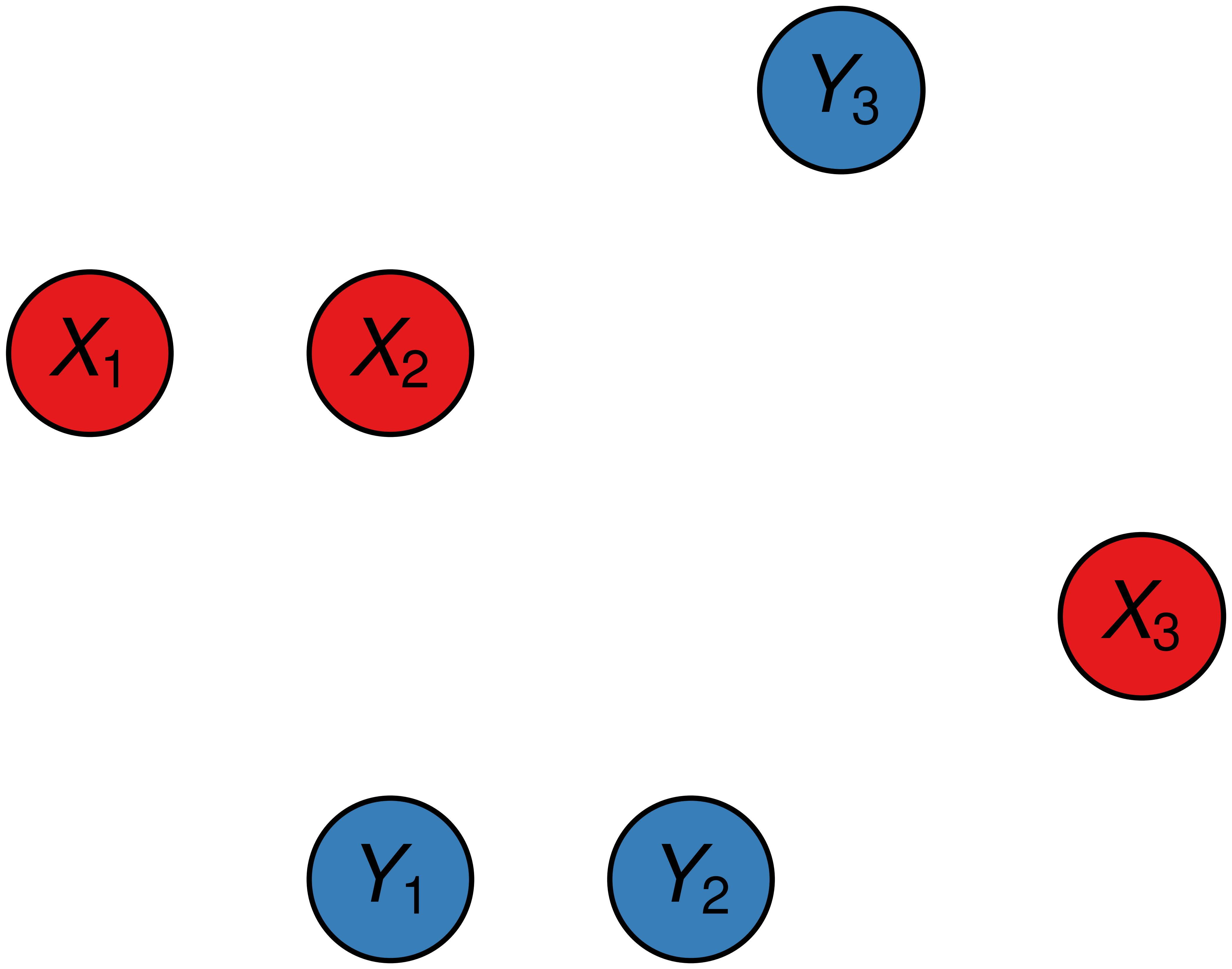


visualization of the 1st phase



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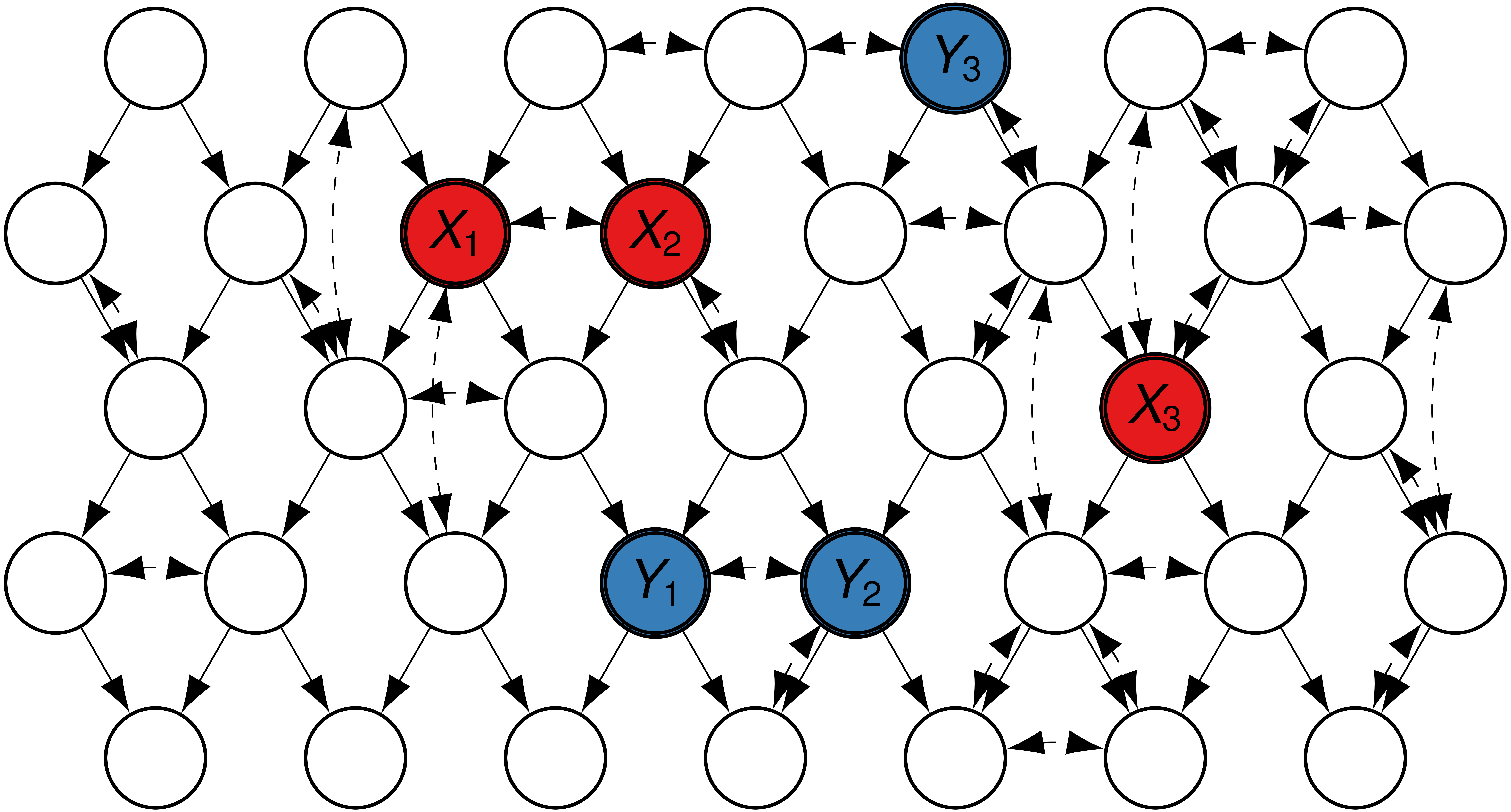

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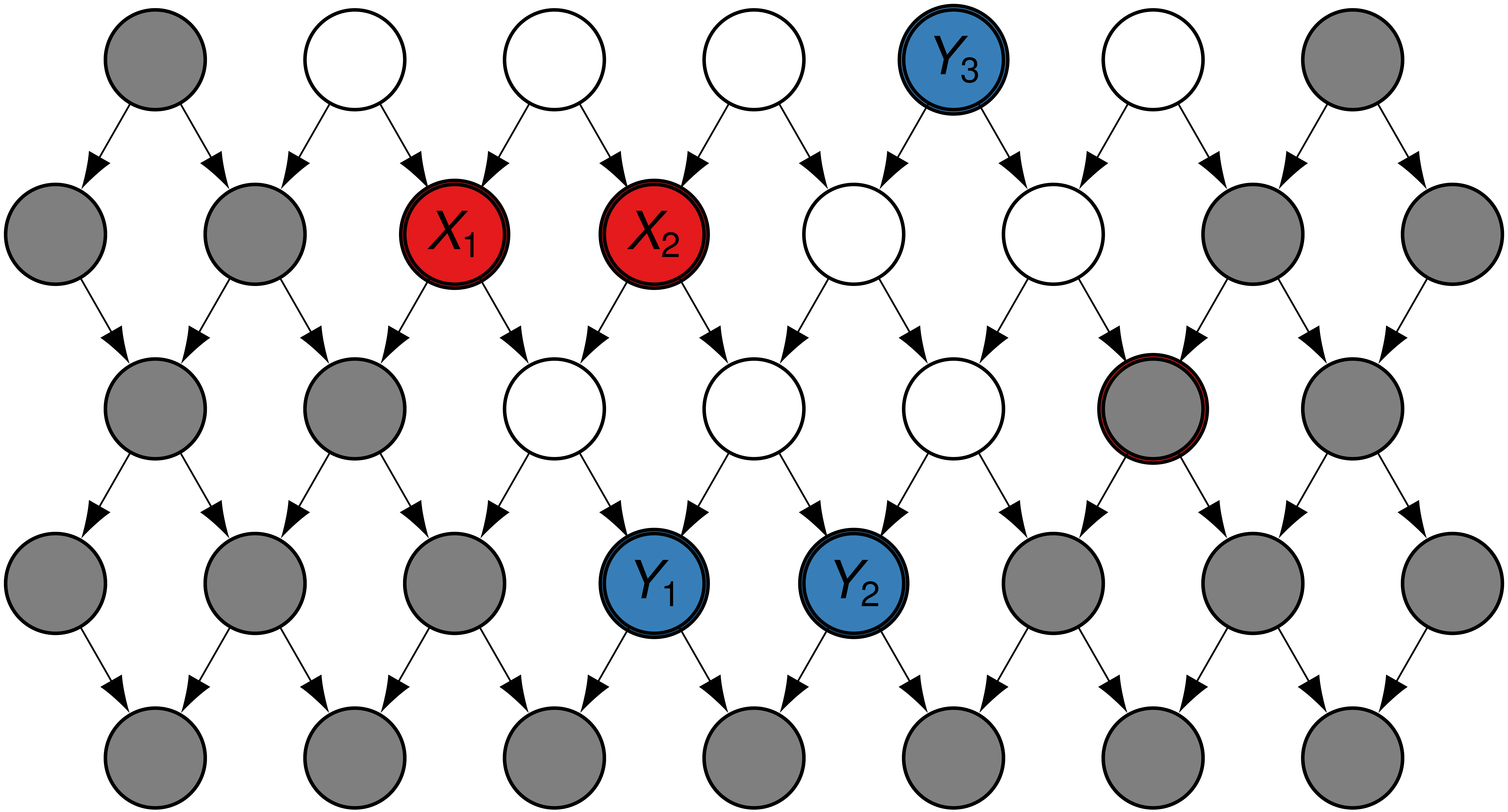

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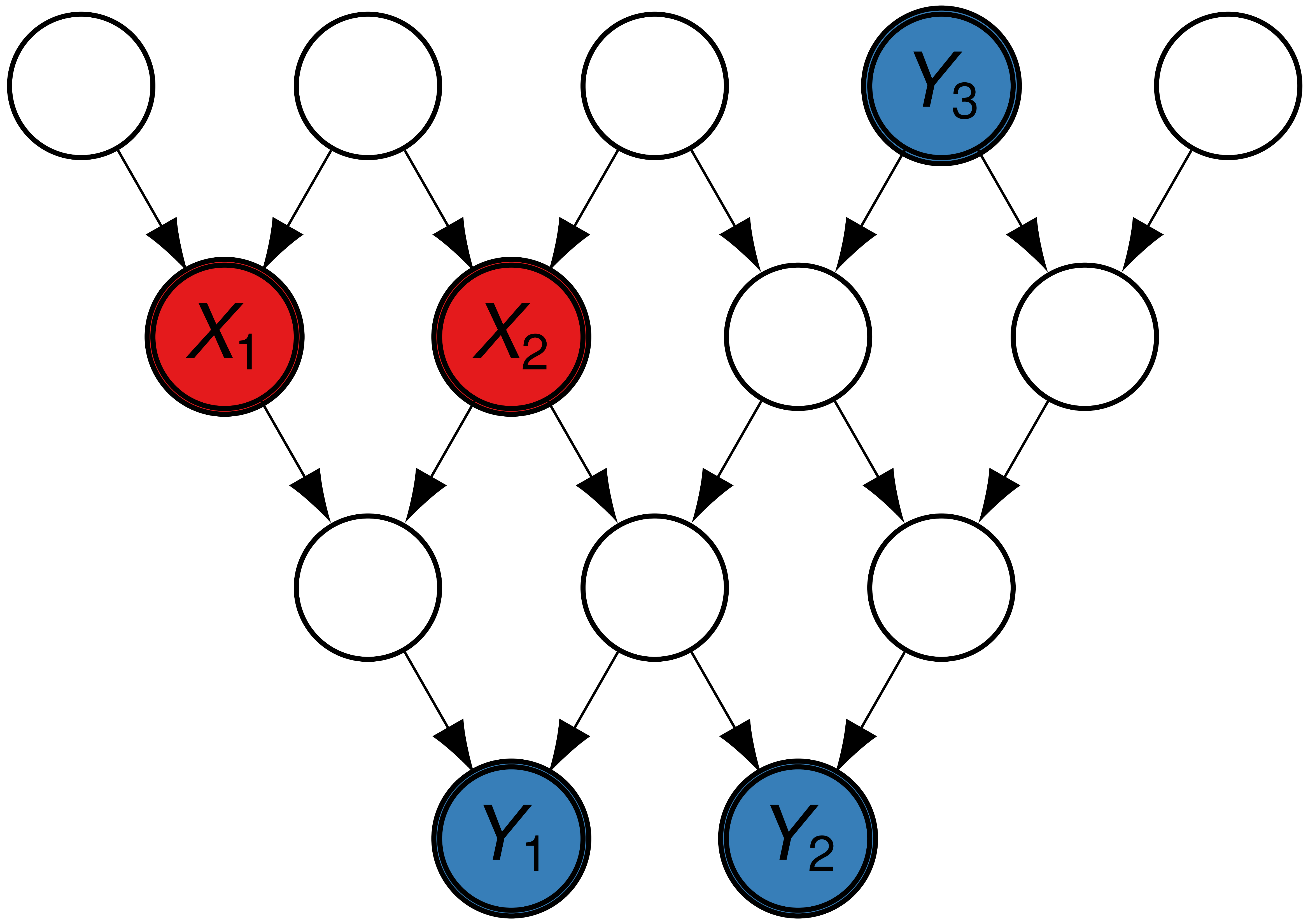

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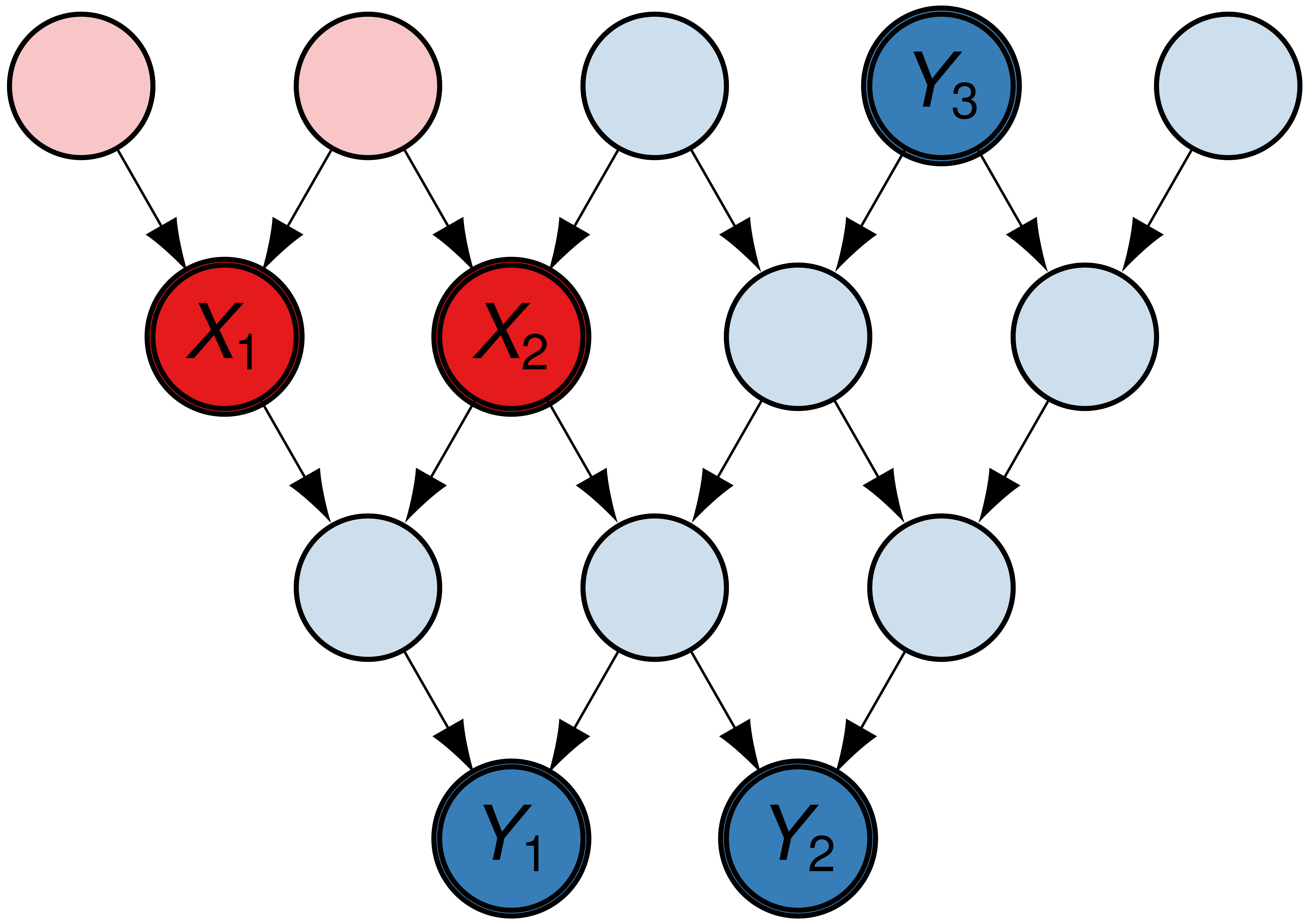

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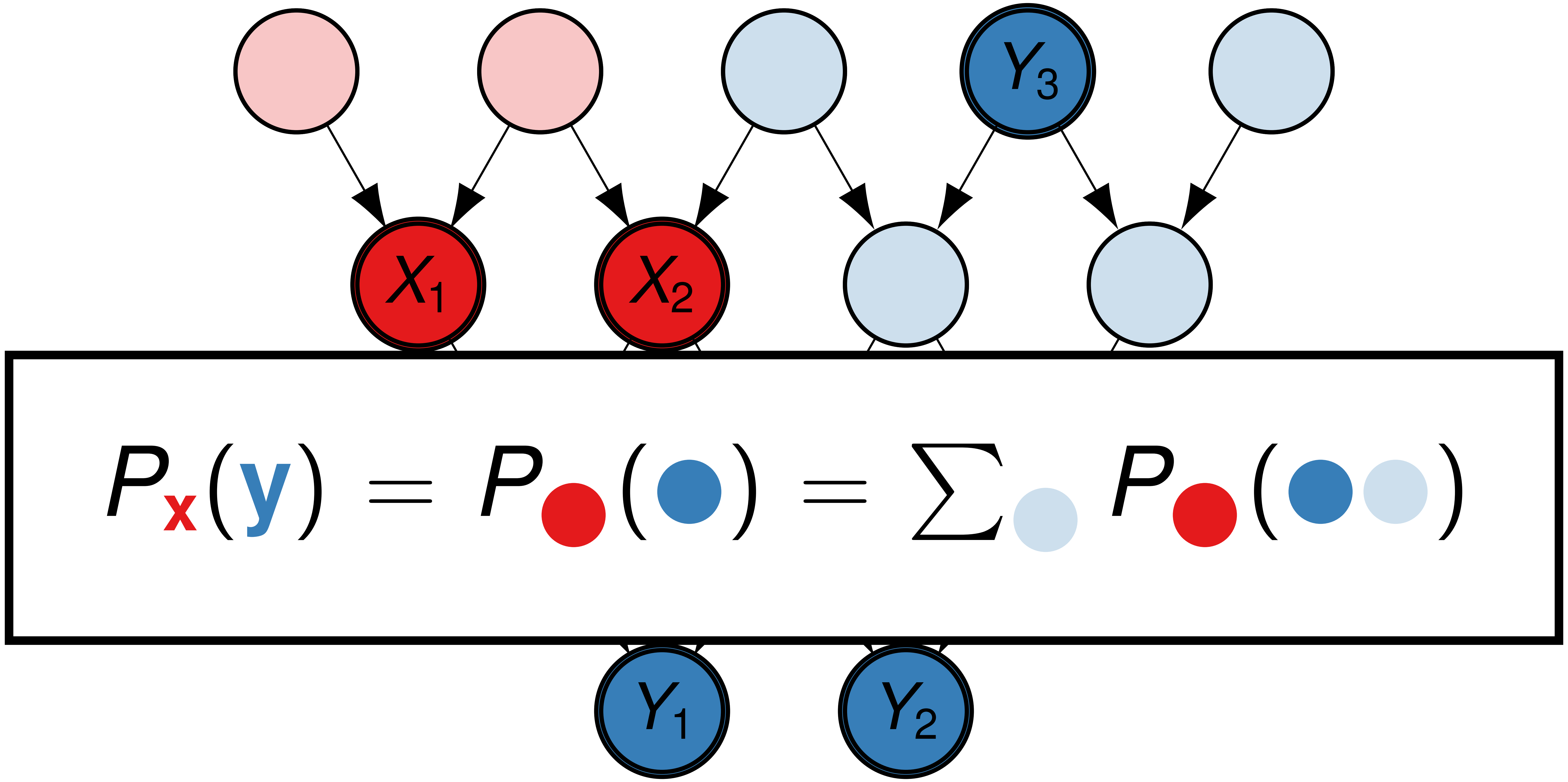

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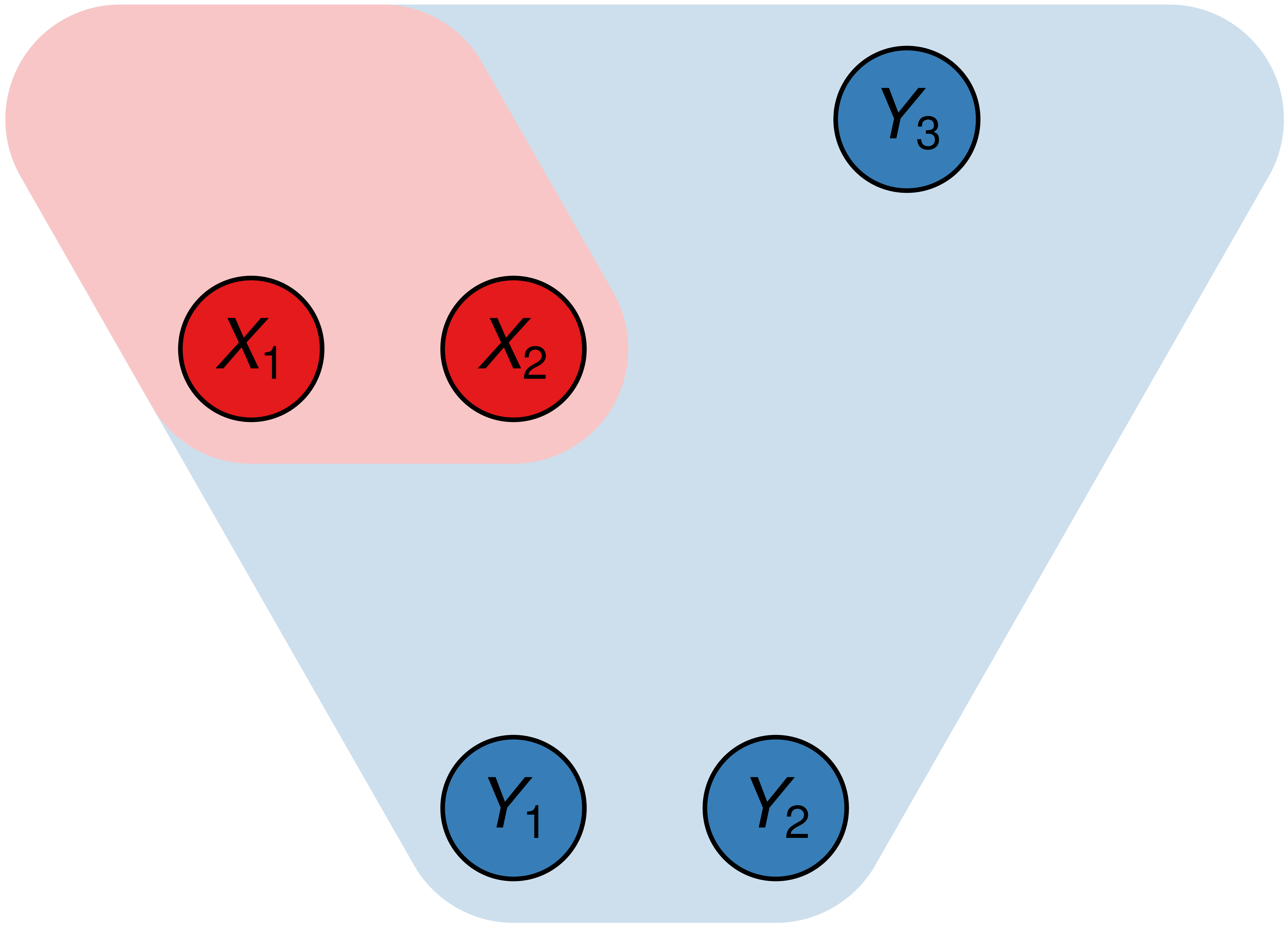

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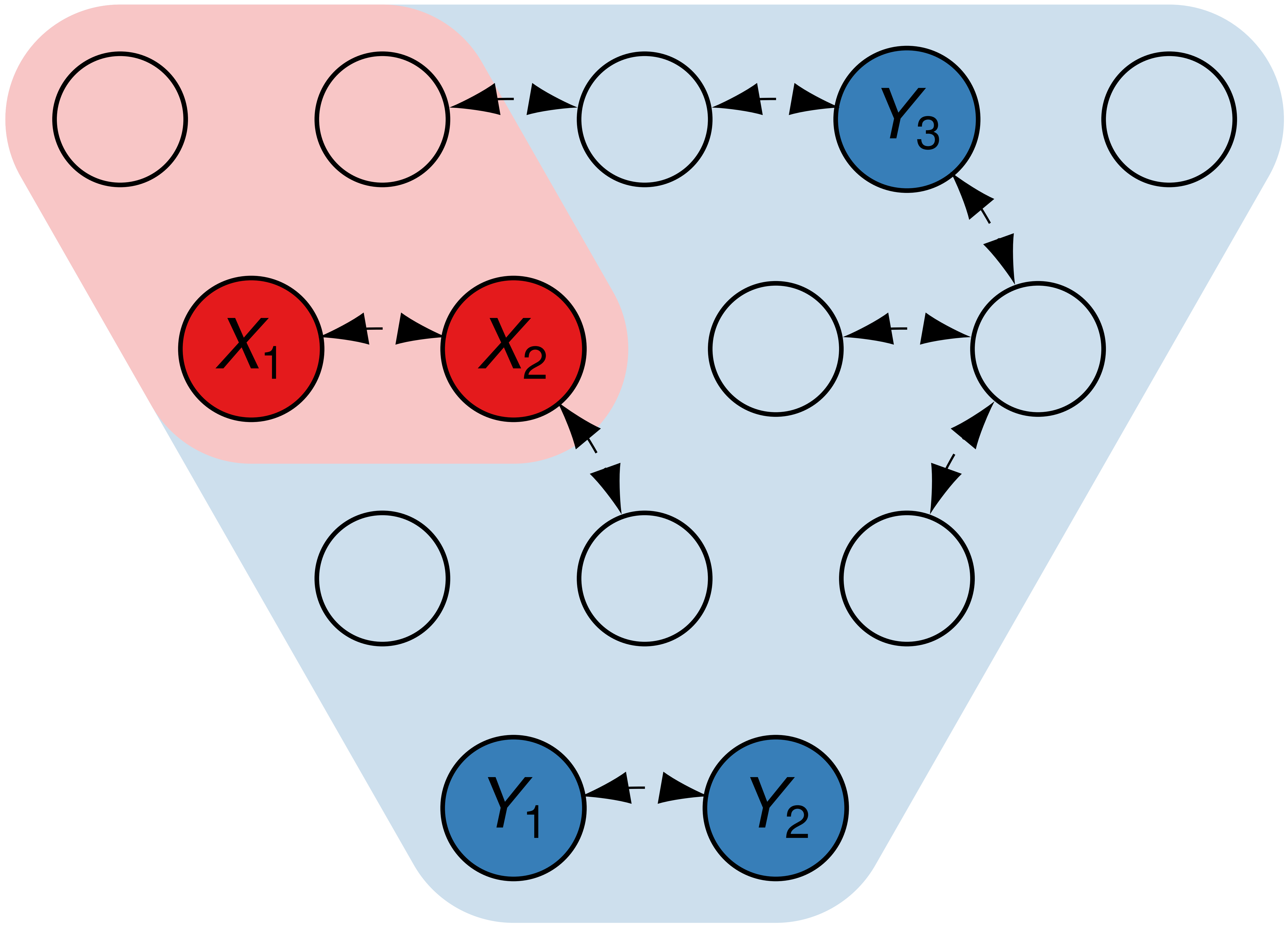

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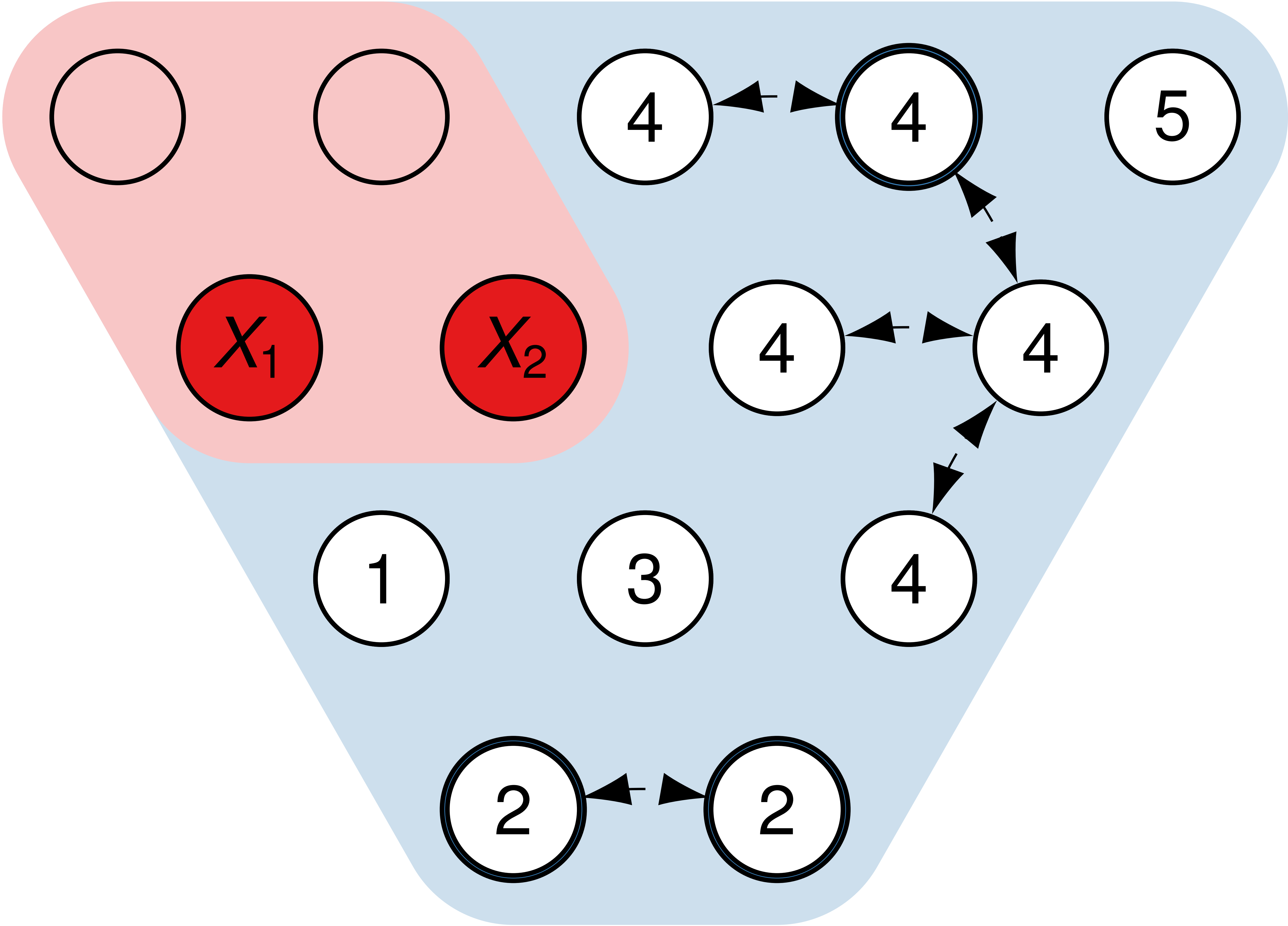

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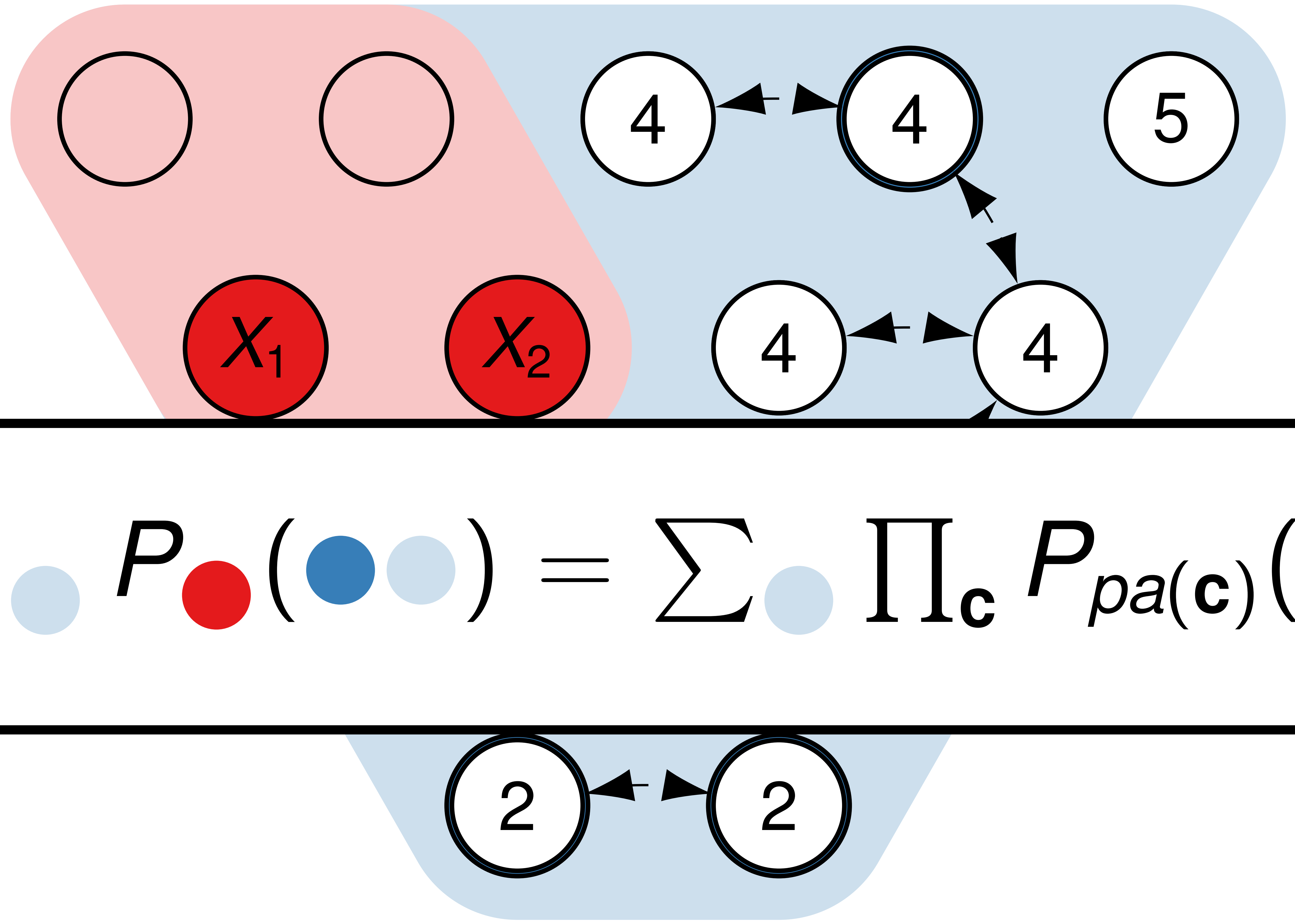
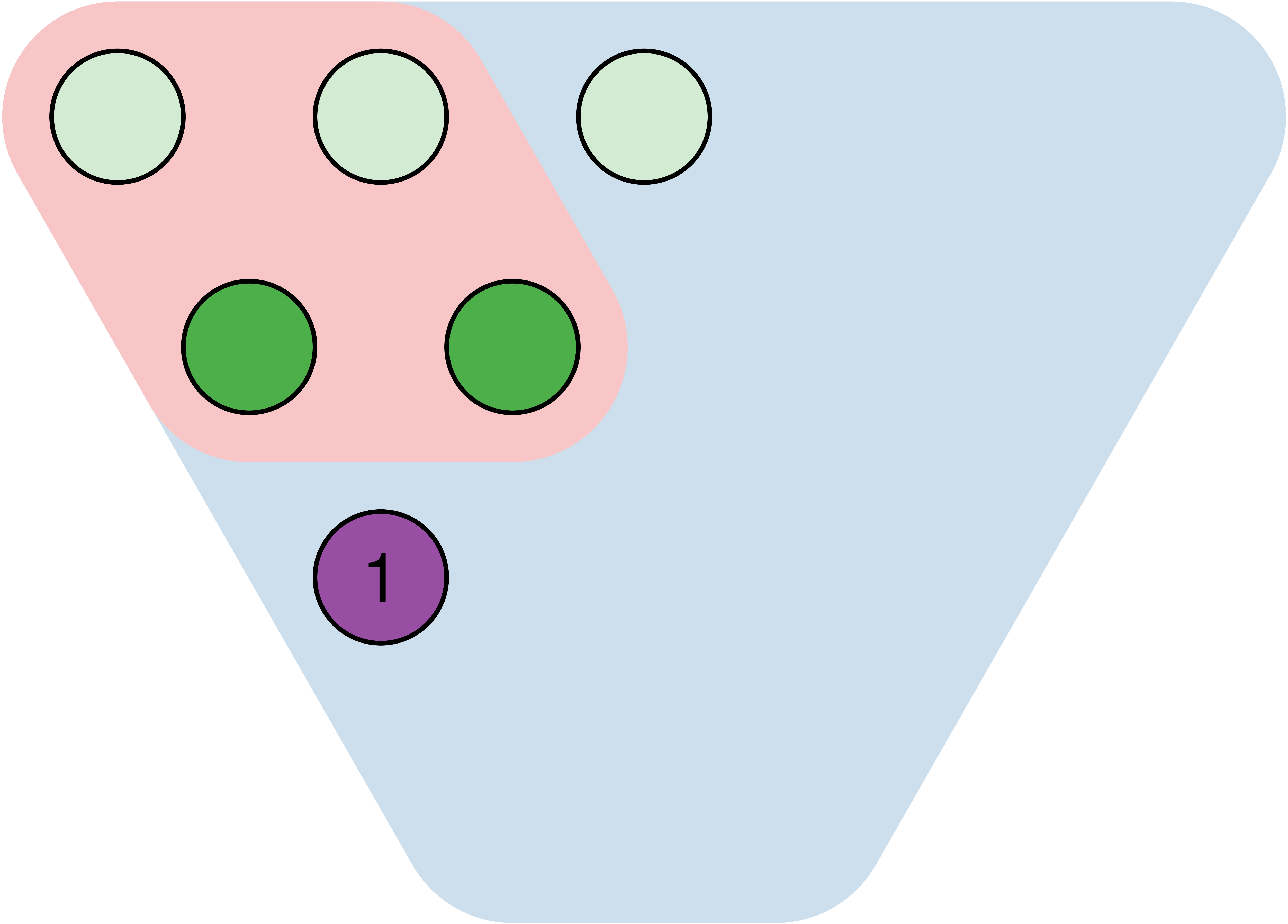


$$\sum_{\bullet} P_{\bullet}(\bullet\bullet) = \sum_{\bullet} \prod_{\mathbf{c}} P_{pa(\mathbf{c})}(\mathbf{c})$$

C: confounded variables

visualization of the 1st phase



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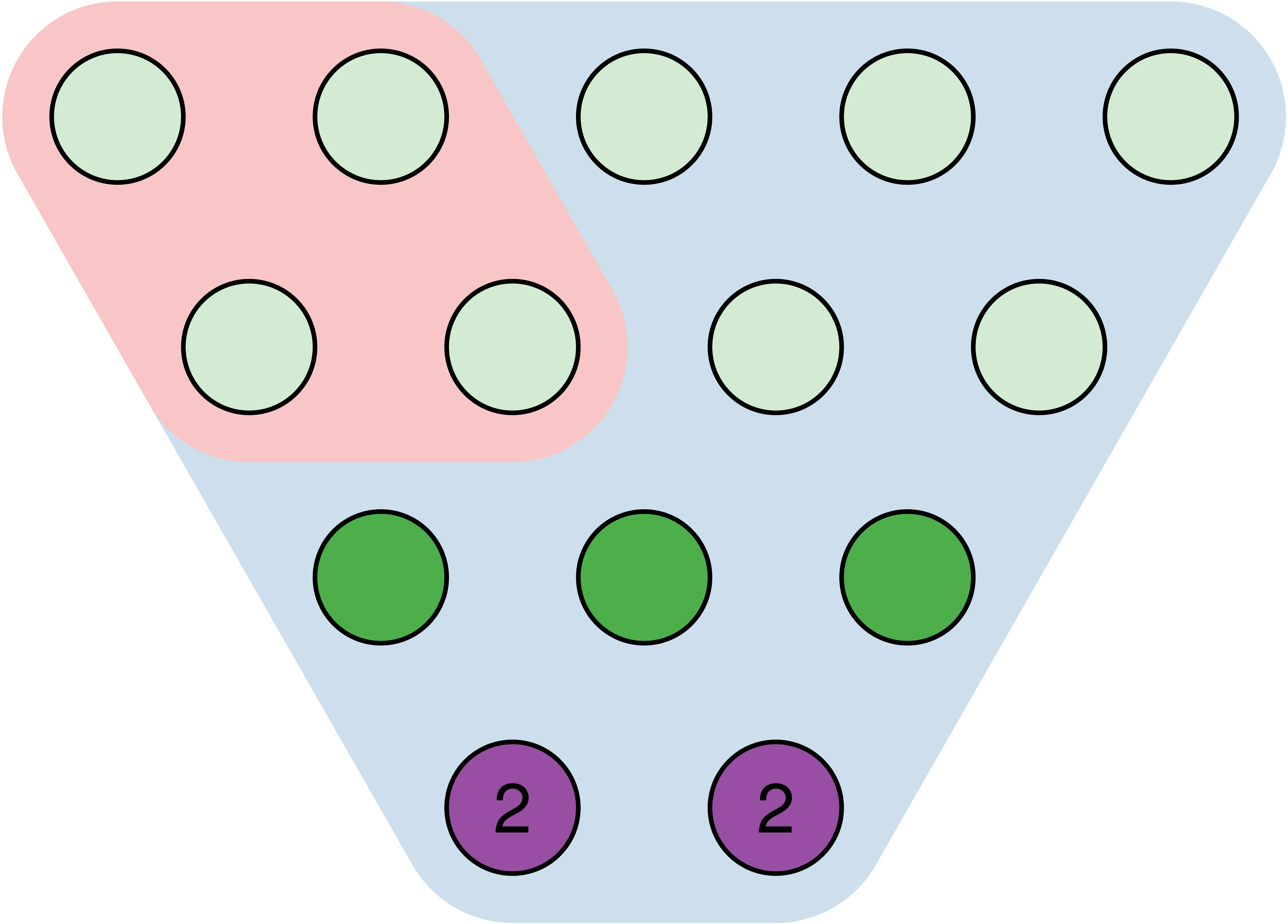

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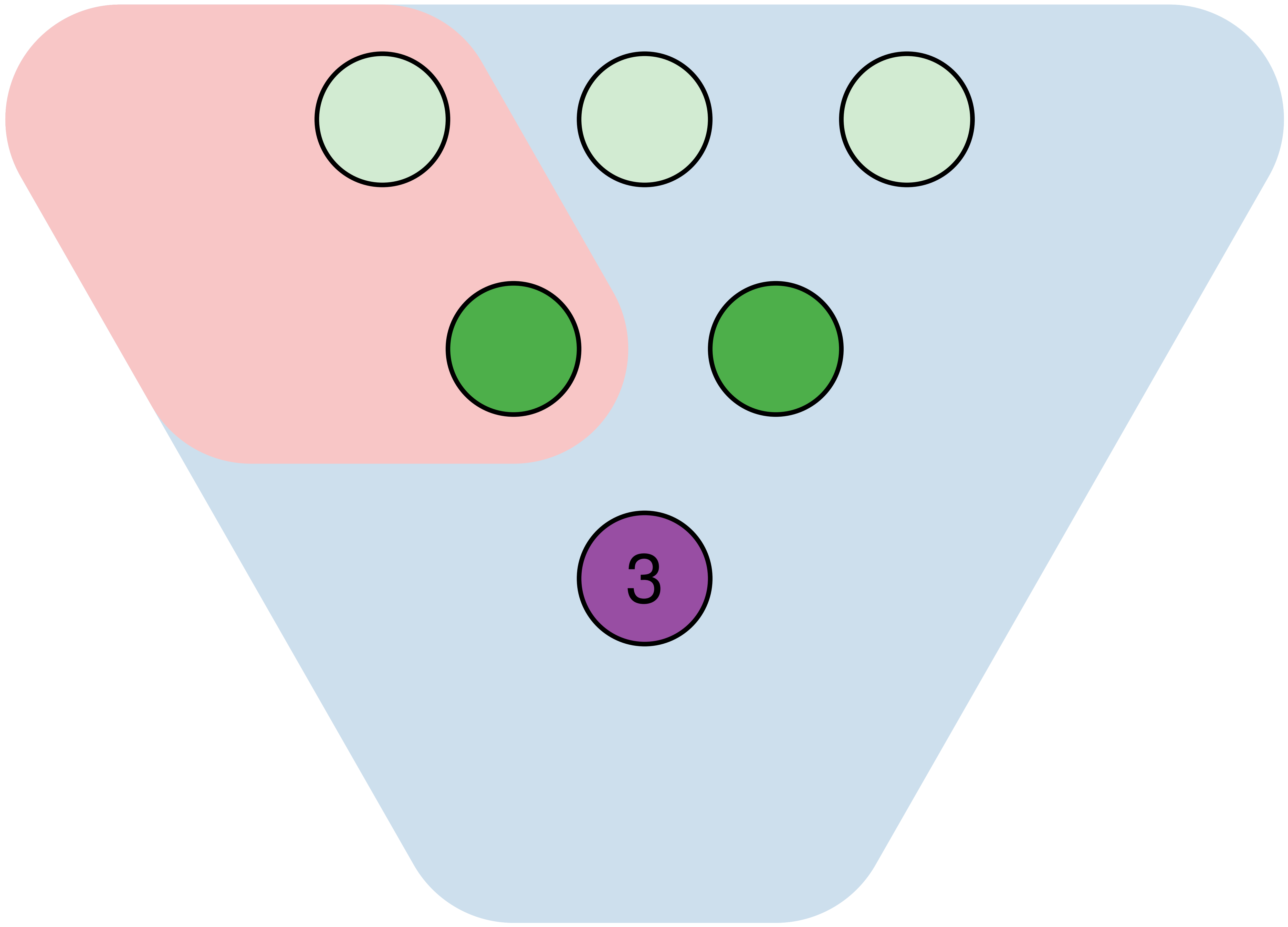

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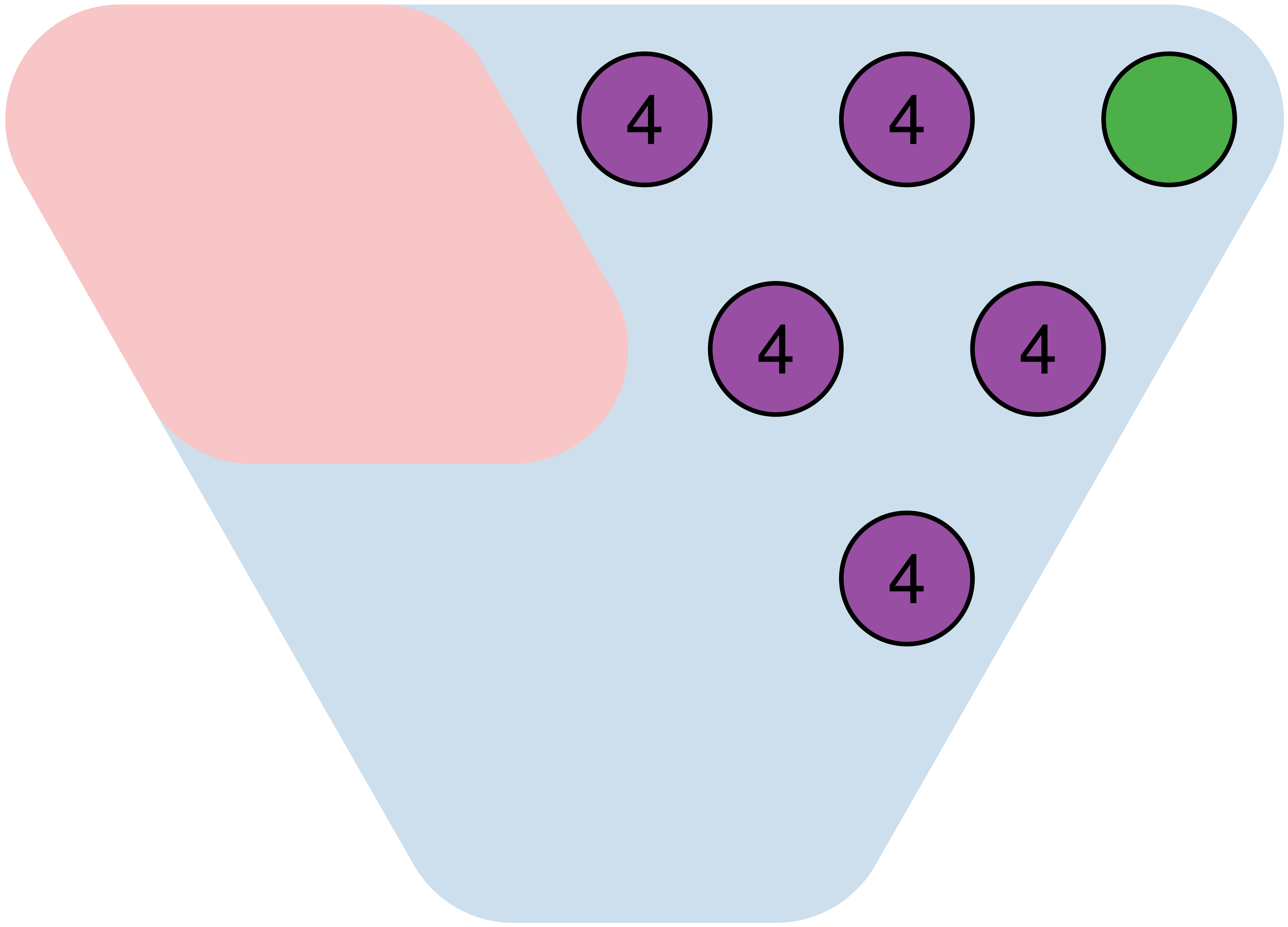

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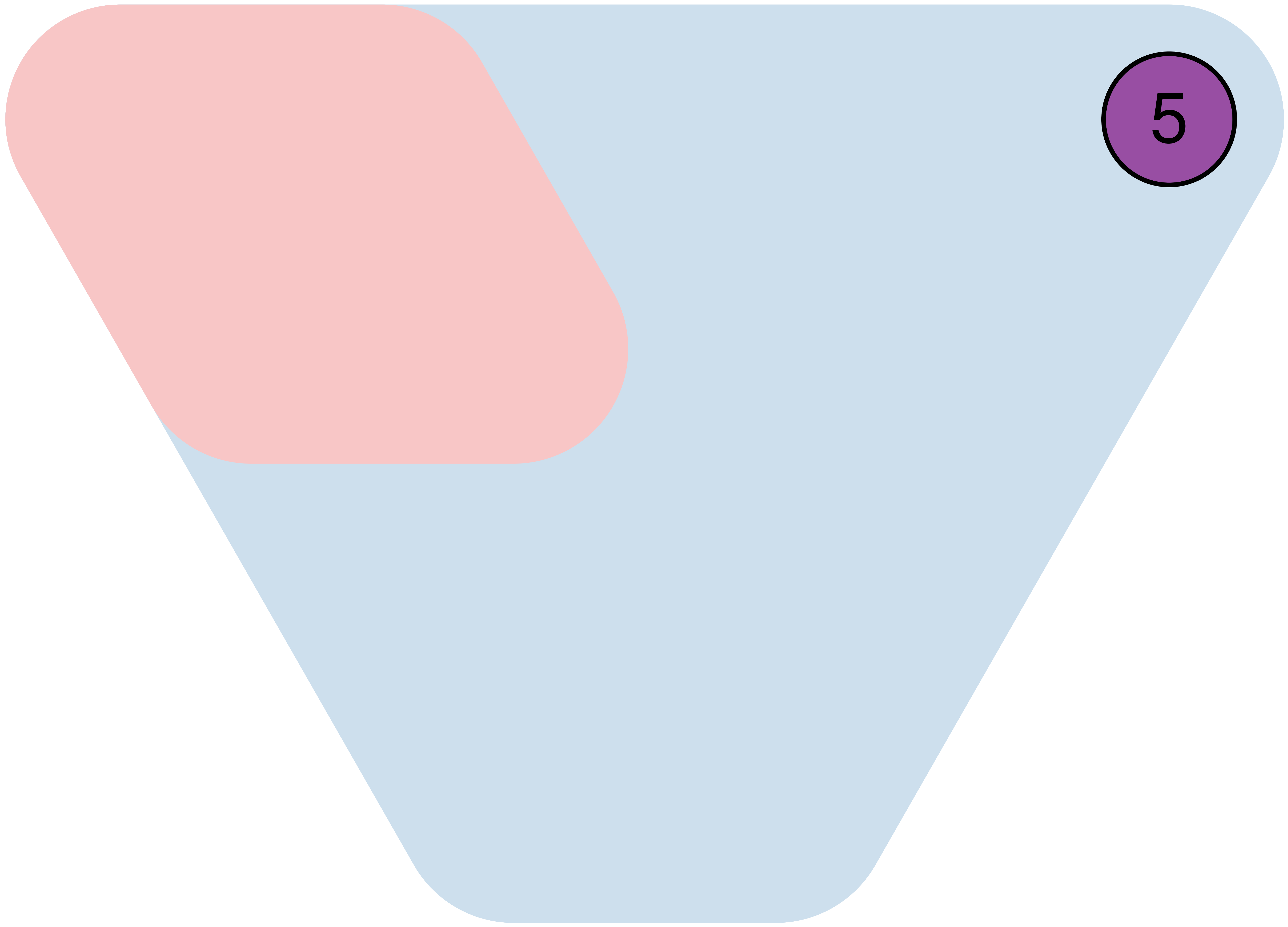

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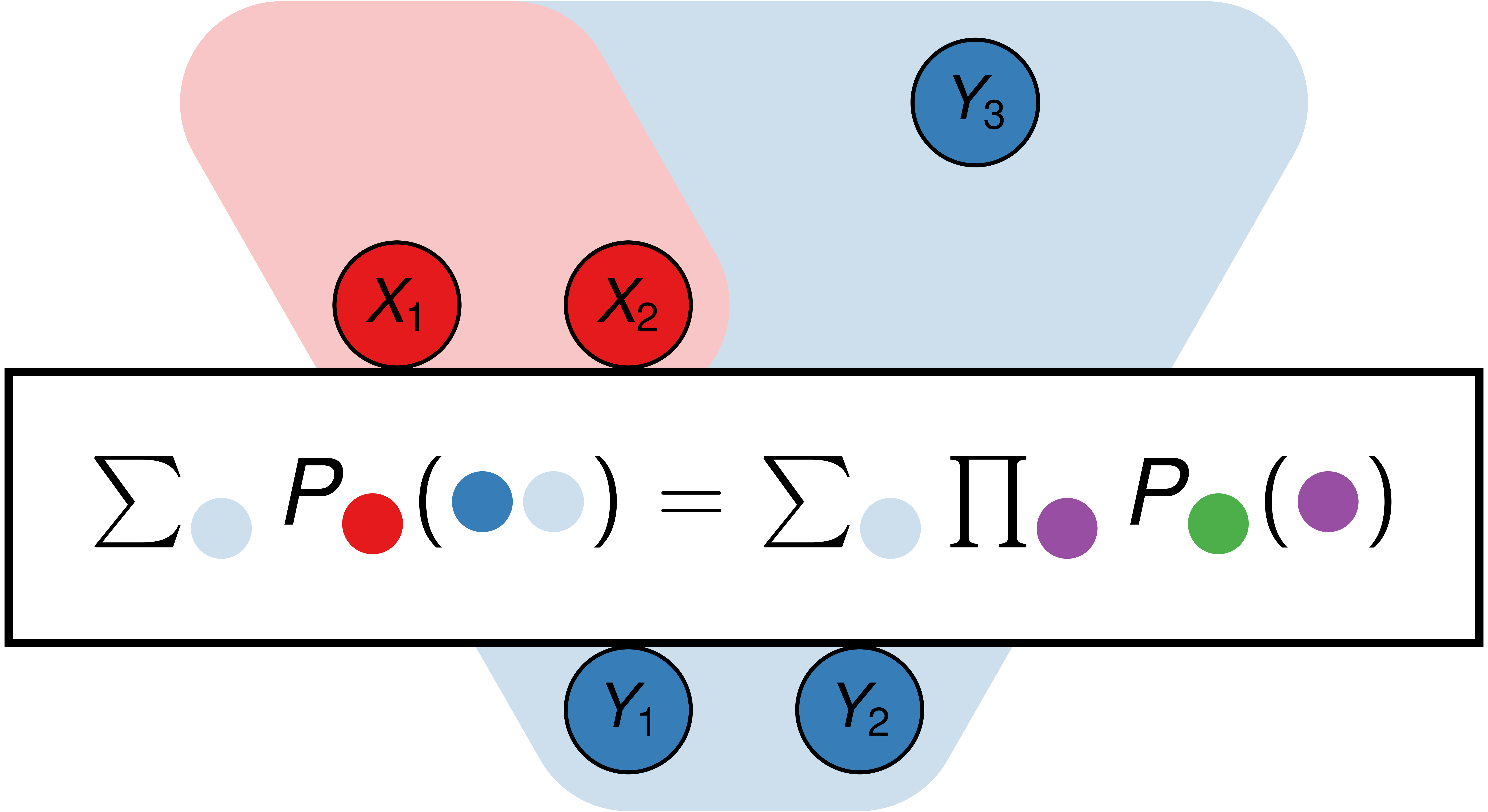


Algorithm for gID (1^{st} phase)

function $\textsc{gID}(\bullet, \bullet, \mathcal{G}, \mathbb{Z})$

if $\exists_{\mathbf{Z} \in \mathbb{Z}} \bullet = \mathbf{Z} \cap \mathbf{V}$ **then** ▷ check whether a matching experiment exists

  return $P_{\mathbf{z} \setminus \mathbf{v}, \bullet}(\bullet)$

if $\mathbf{V} \neq An(\bullet)_{\mathcal{G}}$ **then** ▷ retain only the ancestors of $\bullet$

  return $\textsc{gID}(\bullet, \bullet \cap An(\bullet)_{\mathcal{G}}, \mathcal{G}[An(\bullet)_{\mathcal{G}}], \mathbb{Z})$

if $(\mathbf{W} \leftarrow (\mathbf{V} \setminus \bullet) \setminus An(\bullet)_{\mathcal{G}_{\overline{\bullet}}}) \neq \emptyset$ **then**

  return $\textsc{gID}(\bullet, \bullet \cup \bullet, \mathcal{G}, \mathbb{Z})$ ▷ modify to a maximal intervention

$\mathbb{S} \leftarrow \mathcal{C}(\mathcal{G} \setminus \bullet\bullet)$

if $|\mathbb{S}| > 1$ **then** ▷ factorize into subqueries

  return $\sum_{\bullet} \prod_{\bullet \in \mathbb{S}} \textsc{gID}(\bullet, \bullet\bullet, \mathcal{G}, \mathbb{Z})$

for $\mathbf{Z} \in \mathbb{Z}$ such that $\mathbf{Z} \cap \mathbf{V} \subseteq \bullet\bullet$ **do** ▷ identify with each of available distribution

  return $\textsc{sub-ID}(\bullet, \bullet\bullet \setminus \mathbf{Z}, P_{(\mathbf{z} \setminus \mathbf{v}), \bullet\bullet \cap \mathbf{z}}, \mathcal{G} \setminus (\mathbf{Z} \cap \bullet\bullet))$ **if** not NONE

throw FAIL

sub-ID is a simplified **ID** algorithm [SP'06], which returns NONE if failed.

Algorithm for gID (2nd phase)

function $\textsf{SUB-ID}({\color{purple}\bullet}, {\color{green}\bullet}, Q, \mathcal{G})$

  $\{\mathbf{S}\} \leftarrow \mathcal{C}(\mathcal{G} \setminus {\color{green}\bullet})$

  if ${\color{green}\bullet} = \emptyset$ **then** ▷ check identified

    return $\sum_{\mathbf{v} \setminus {\color{purple}\bullet}} Q(\mathbf{v})$

  if $\mathbf{V} \neq An({\color{purple}\bullet})_{\mathcal{G}}$ **then** ▷ retain only the ancestors of ${\color{purple}\bullet}$

    return $\textsf{SUB-ID}({\color{purple}\bullet}, {\color{green}\bullet} \cap An({\color{purple}\bullet})_{\mathcal{G}}, \sum_{\mathbf{v} \setminus An({\color{purple}\bullet})_{\mathcal{G}}} Q, \mathcal{G}[An({\color{purple}\bullet})_{\mathcal{G}}])$

  if $\mathcal{C}(\mathcal{G}) = \mathbf{V}$ **then** ▷ check the existence of a hedge

    return NONE

  if $\mathbf{S} \in \mathcal{C}(\mathcal{G})$ **then** ▷ check identifiable

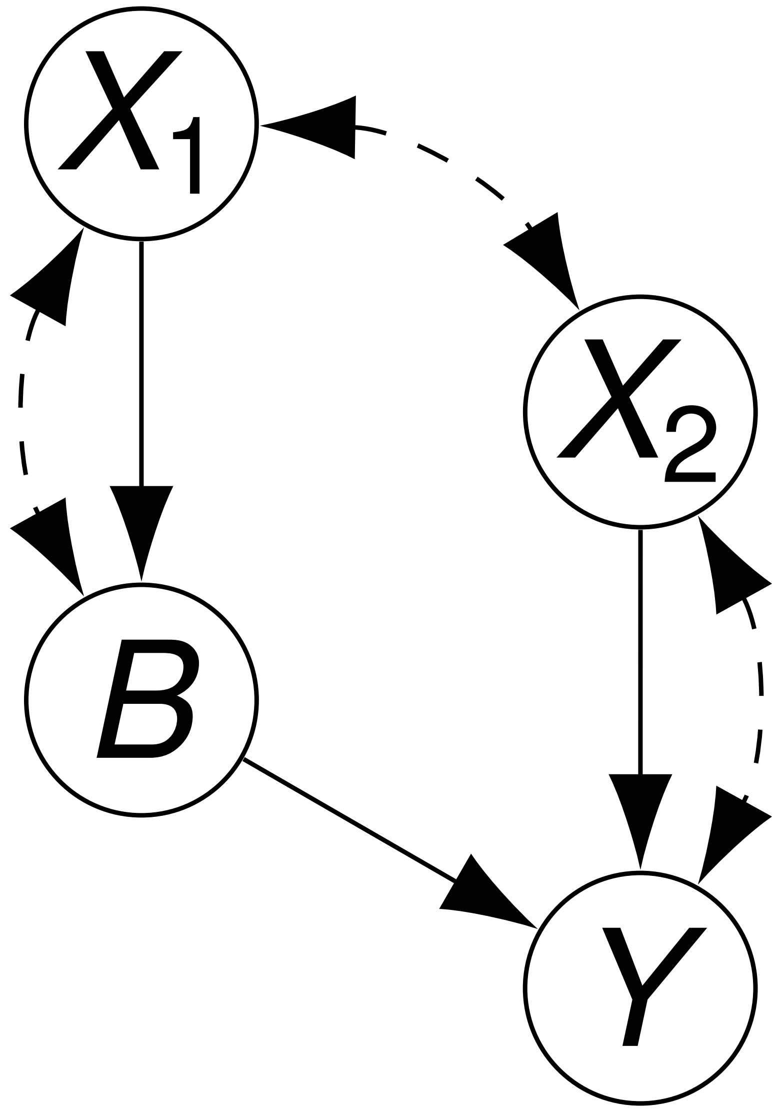
    return $\sum_{\mathbf{s} \setminus {\color{purple}\bullet}} \prod_{V_i \in {\color{purple}\bullet}} Q(v_i | \mathbf{v}_{\pi}^{(i-1)})$.

  if $\mathbf{S} \subsetneq \mathbf{S}' \in \mathcal{C}(\mathcal{G})$ **then** ▷ modify input (query, distribution, and graph)

    return $\textsf{SUB-ID}({\color{purple}\bullet}, {\color{green}\bullet} \cap \mathbf{S}', \prod_{V_i \in \mathbf{S}'} Q(V_i | \mathbf{V}_{\pi}^{(i-1)} \cap \mathbf{S}', \mathbf{v}_{\pi}^{(i-1)} \setminus \mathbf{S}'), \mathbf{S}')$

Example: Durg-Drug Interactions

Y: heart disease, B: blood pressure, X_1, X_2: drugs



$$P_{x_1, x_2}(y)$$

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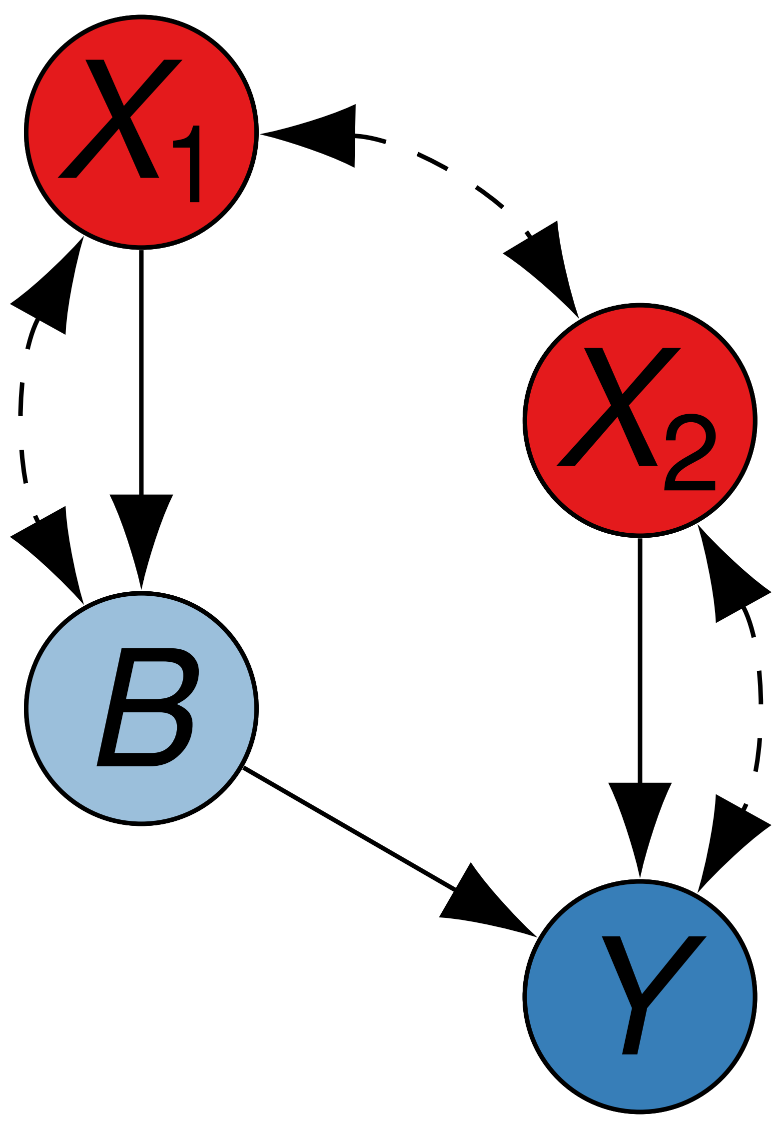

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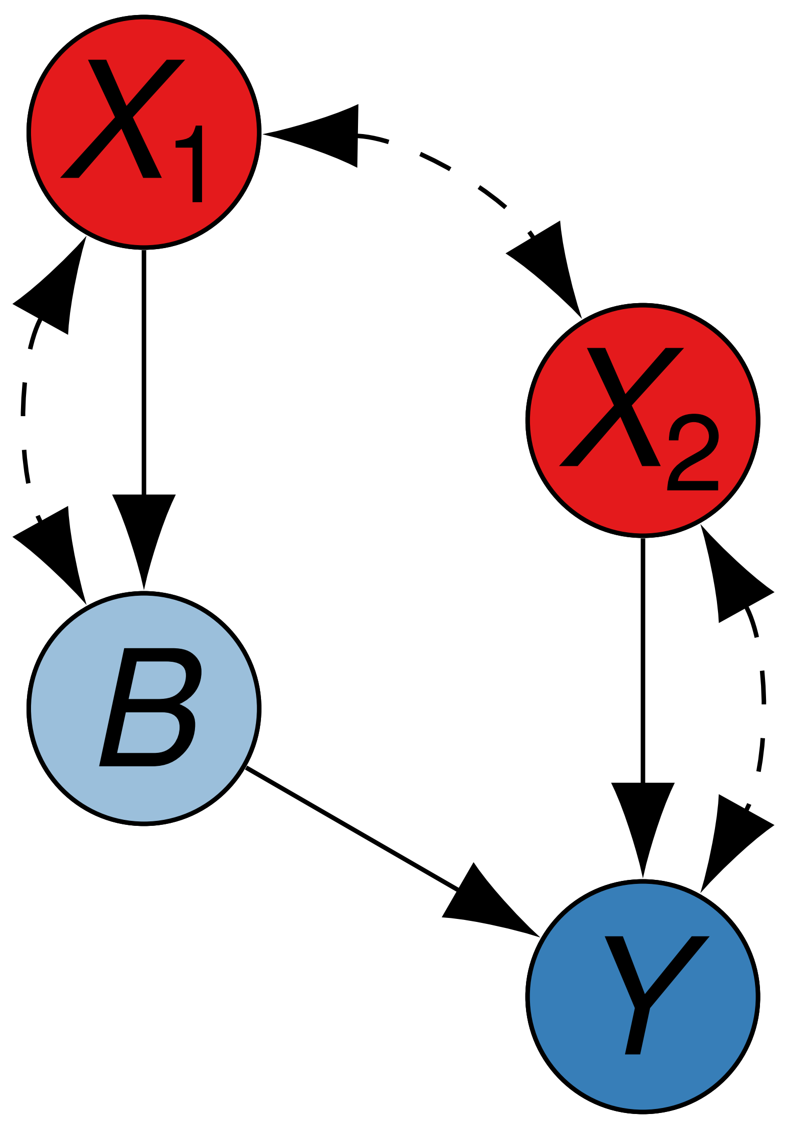

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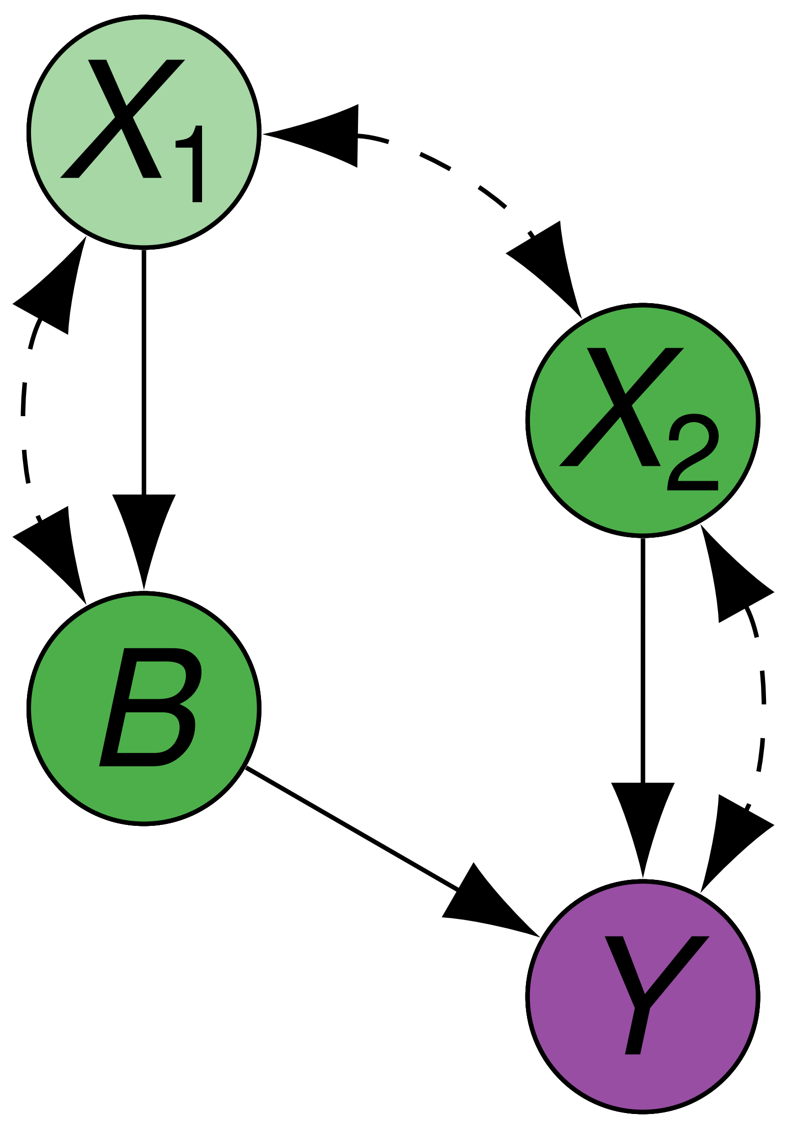

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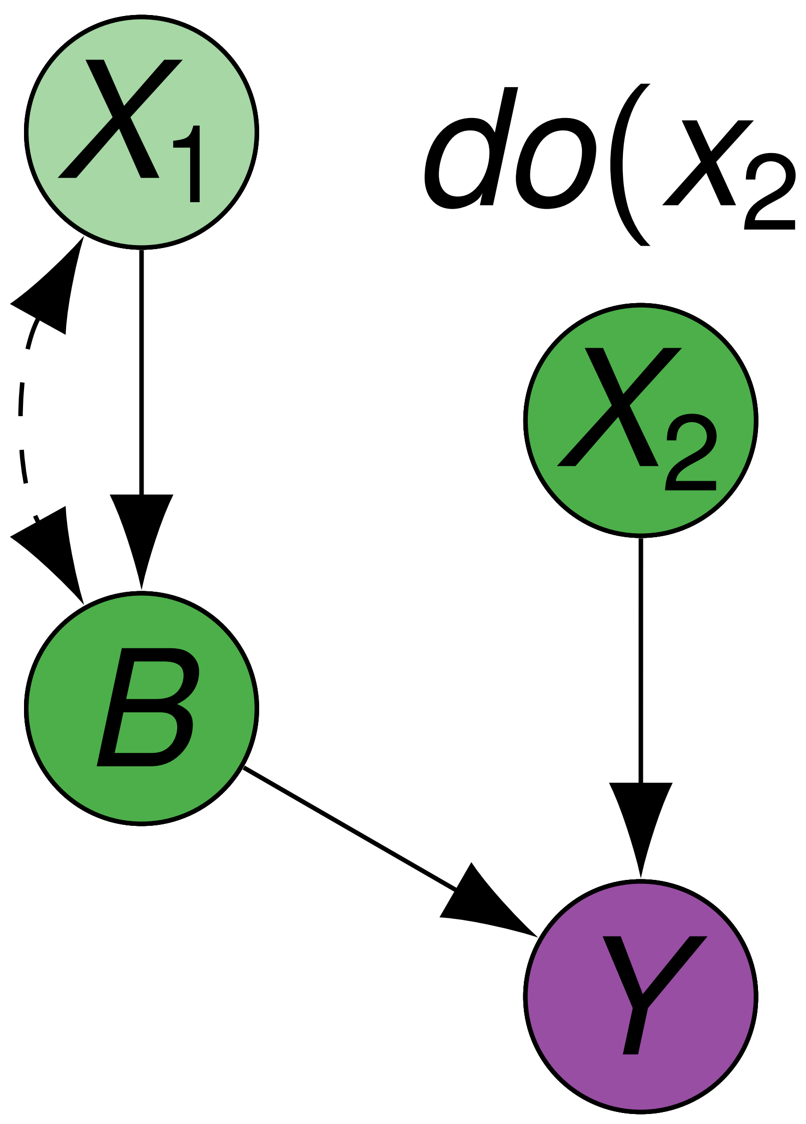

$do(x_2)$

$$
\begin{aligned}
P_{x_1,x_2}(y) &= \sum_b P_{x_1,x_2}(y,b) \\
&= \sum_b P_{x_1}(b) P_{x_2,b}(y) \\
&= \sum_b P_{x_1}(b) P_{x_2}(y|b)
\end{aligned}
$$

non-g-identifiability

– the failure condition & a prohibiting structure

Proving Non-g-Identifiability

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Proving Non-g-Identifiability

To prove $P_{\mathbf{x}}(\mathbf{y})$ is **not g-identifiable** from $\mathbb{Z}$ in $\mathcal{G}$, we construct two causal models $\mathcal{M}_1$ and $\mathcal{M}_2$ compatible with $\mathcal{G}$

$$\text{SCM } \mathcal{M}^1 \longrightarrow \mathcal{G} \longleftarrow \text{SCM } \mathcal{M}^2$$

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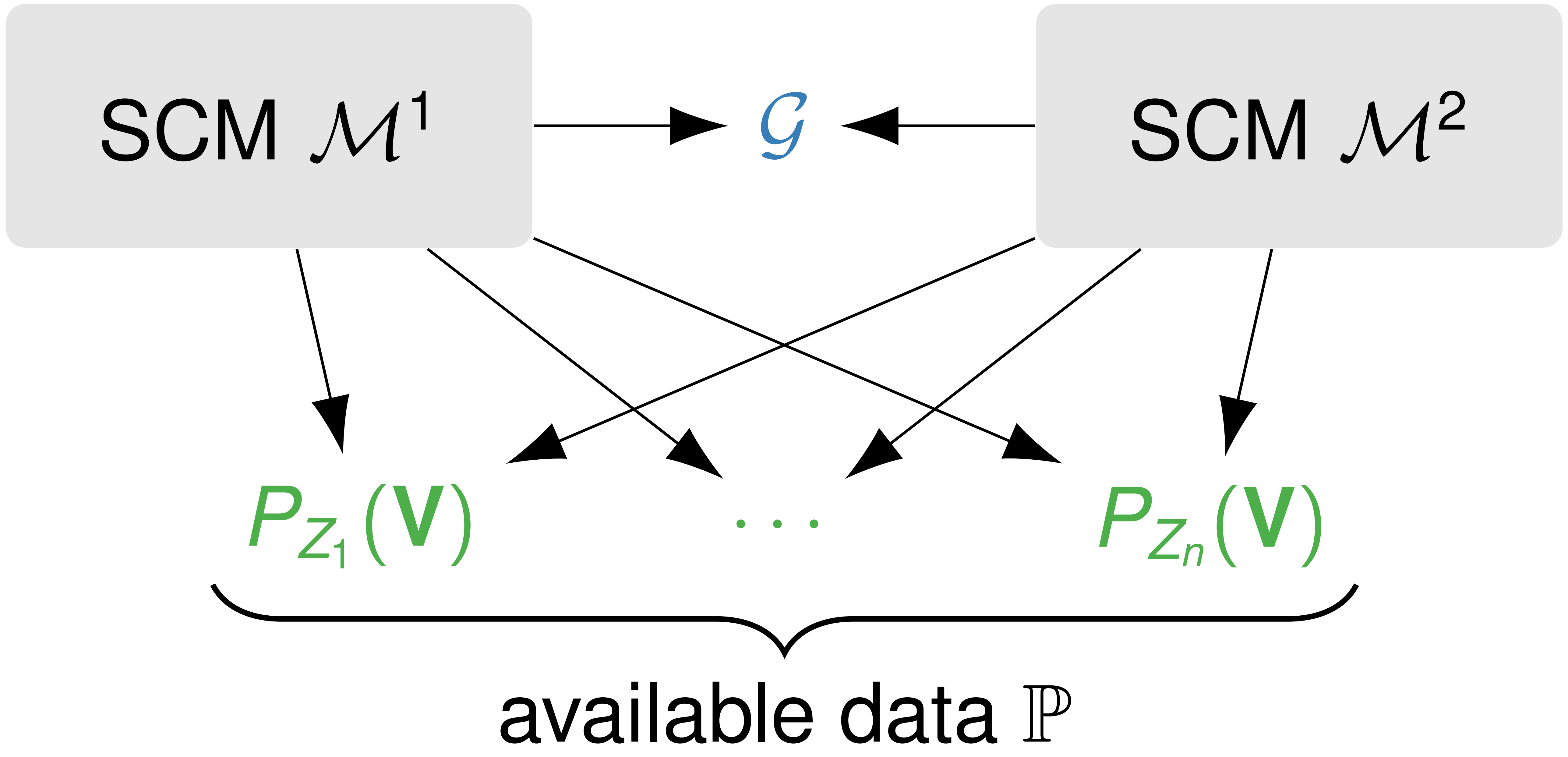
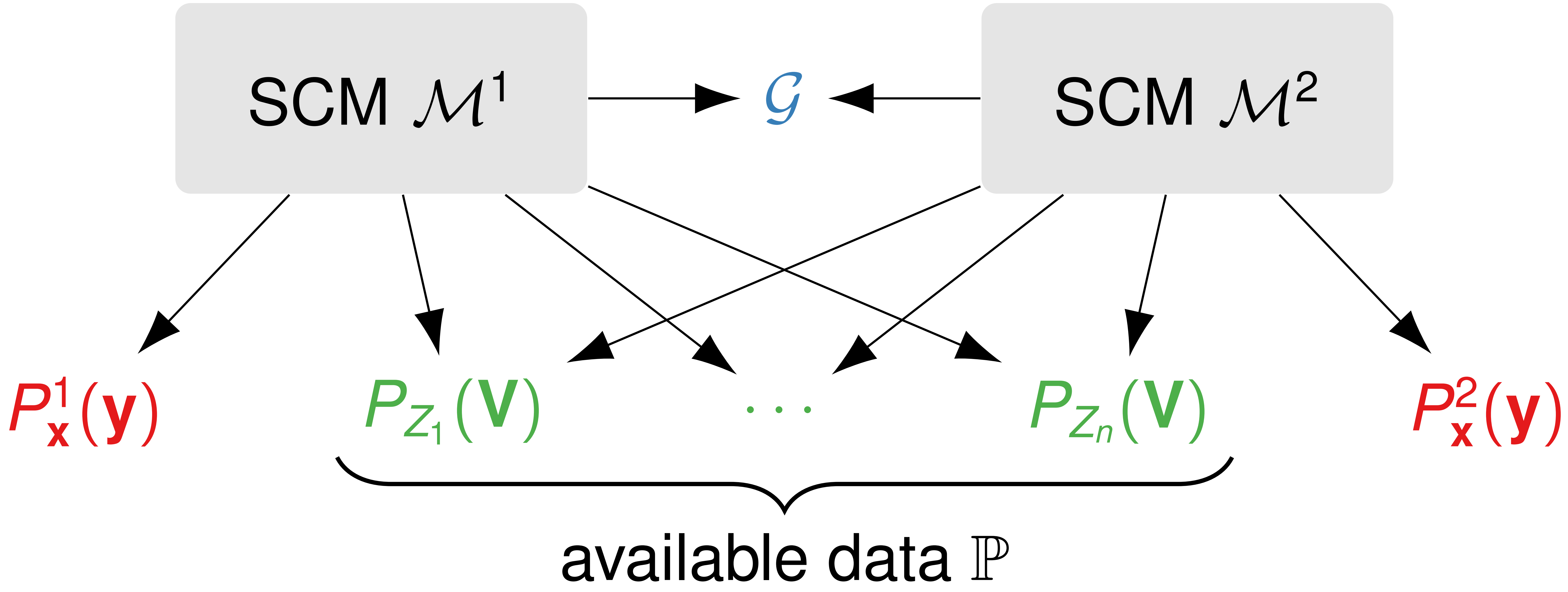
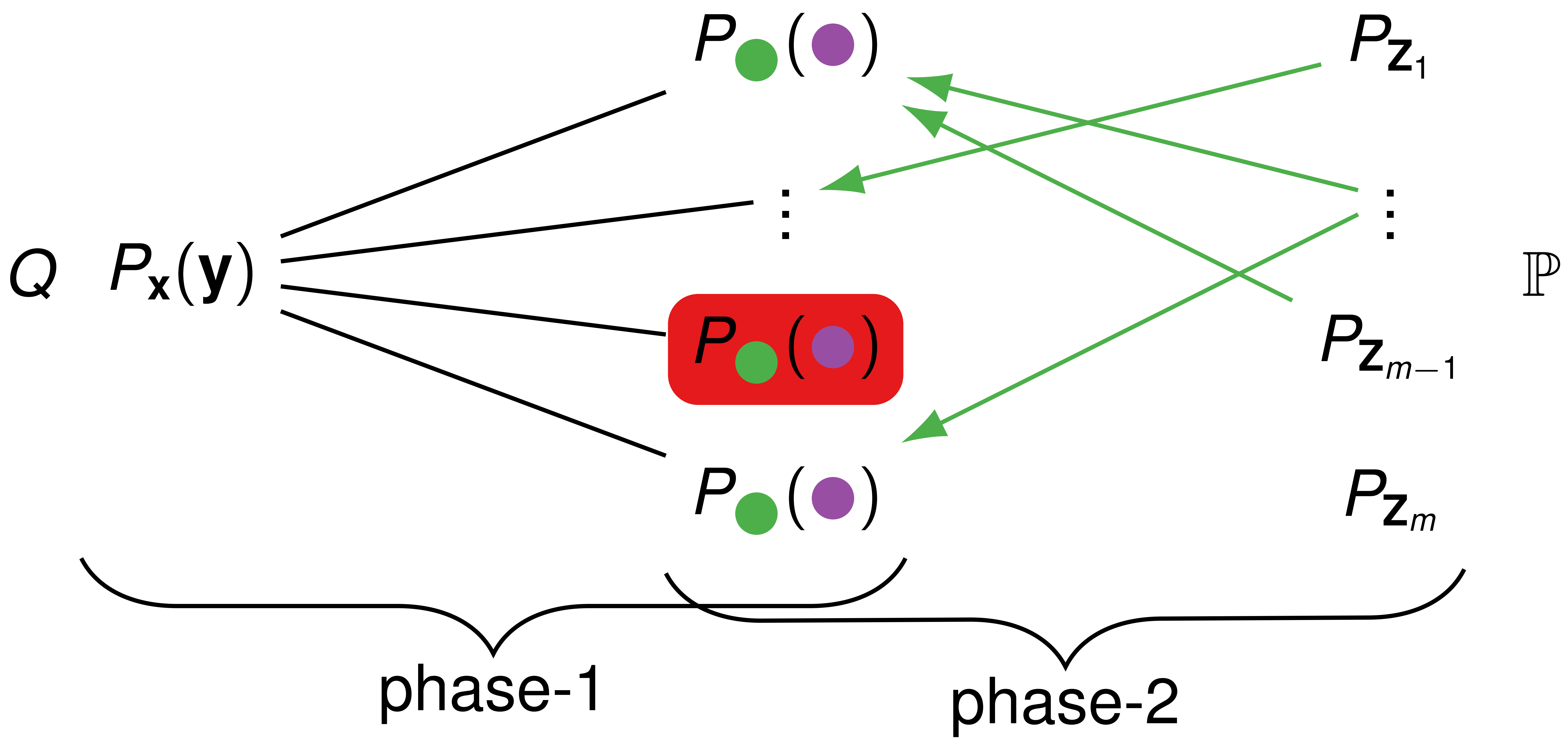


Proving Non-g-Identifiability

To prove $P_{\mathbf{x}}(\mathbf{y})$ is **not g-identifiable** from $\mathbb{Z}$ in $\mathcal{G}$, we construct two causal models $\mathcal{M}_1$ and $\mathcal{M}_2$ compatible with $\mathcal{G}$ such that $P^1_{\mathbf{z}}(\mathbf{v}) = P^2_{\mathbf{z}}(\mathbf{v})$ for all $\mathbf{Z} \in \mathbb{Z}, \mathbf{z} \in \mathfrak{X}_{\mathbf{Z}}$, but $P^1_{\mathbf{x}}(\mathbf{y}) \neq P^2_{\mathbf{x}}(\mathbf{y})$.



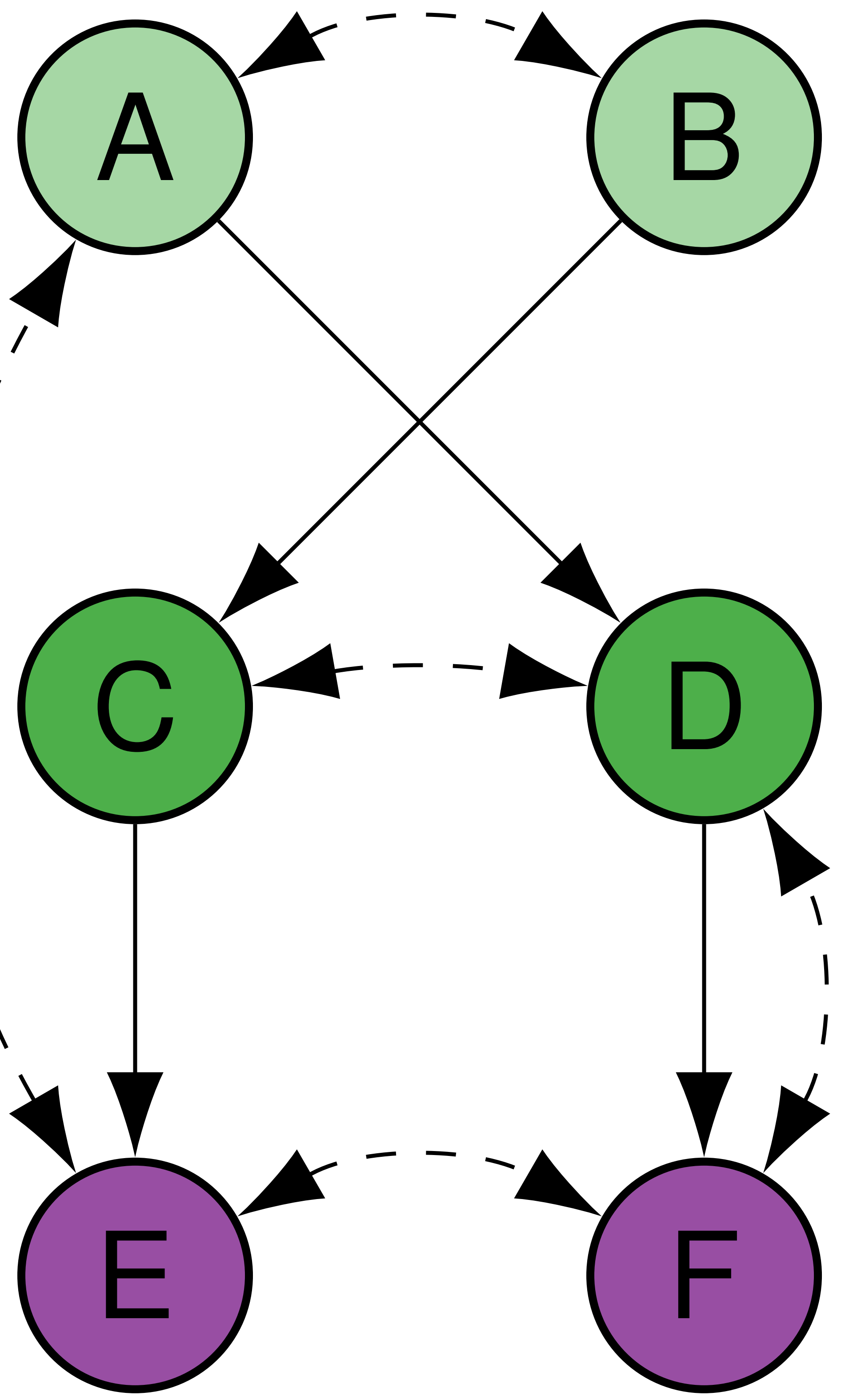
Recall the failed factor ...



$\exists P_{\bullet}(\bullet)$ such that $\forall P_{\mathbf{z}_i} \in \mathbb{P}$ fails

$P_{\bullet}(\bullet)$ versus $P_{\mathbf{z}_i} \in \mathbb{P}$ (phase-2)

There are 3 situations in identifying a factor $P_{\bullet}(\bullet)$ with a distribution $P_{\mathbf{z}_i} \in \mathbb{P}$.



$P_{\color{green}\bullet}({\color{purple}\bullet})$ versus $P_{\mathbf{z}_i} \in \mathbb{P}$ (phase-2)

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- ✓ (the **good**) identified,
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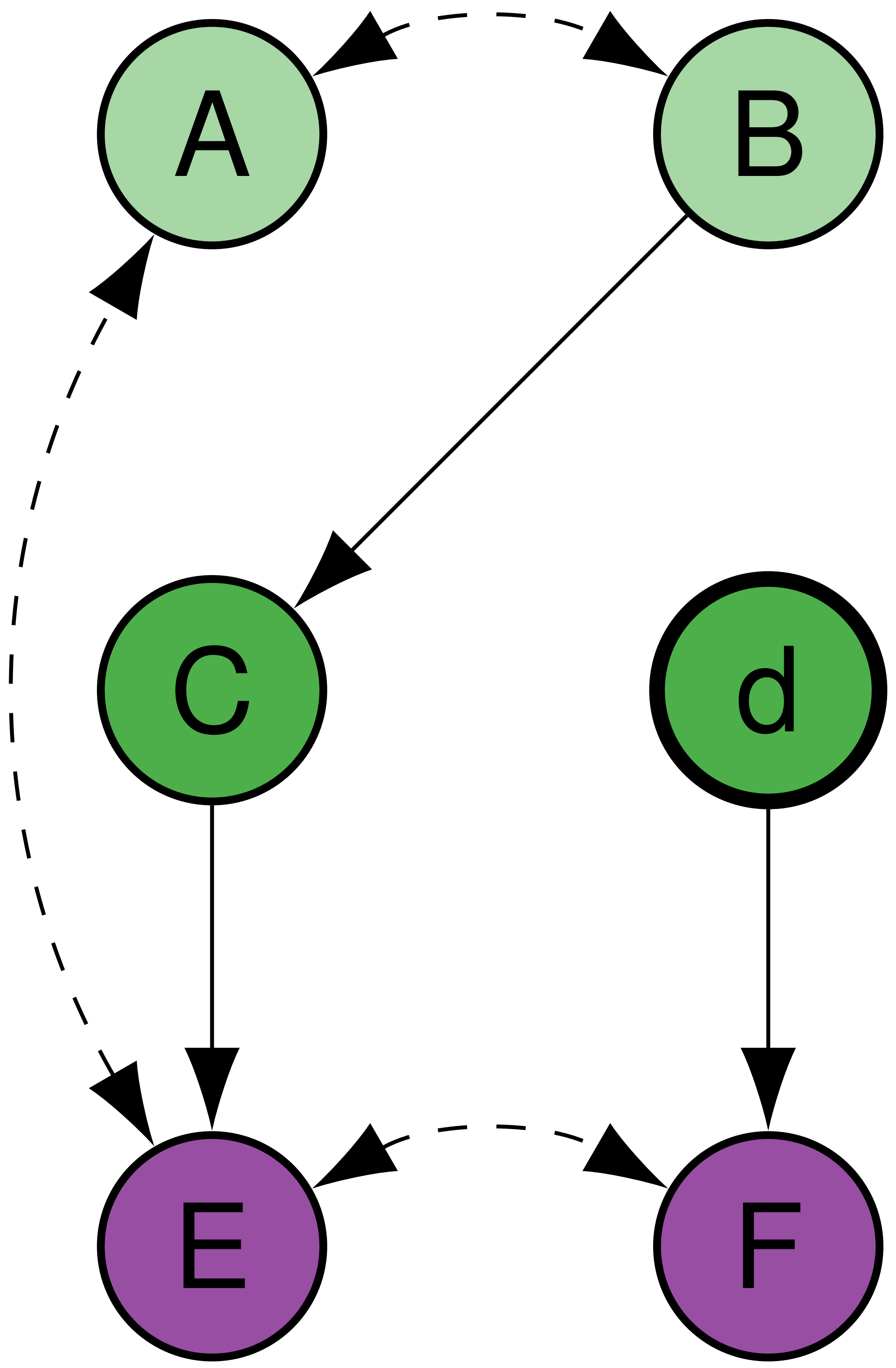

the original order should be: the good, the bad, and the ugly.

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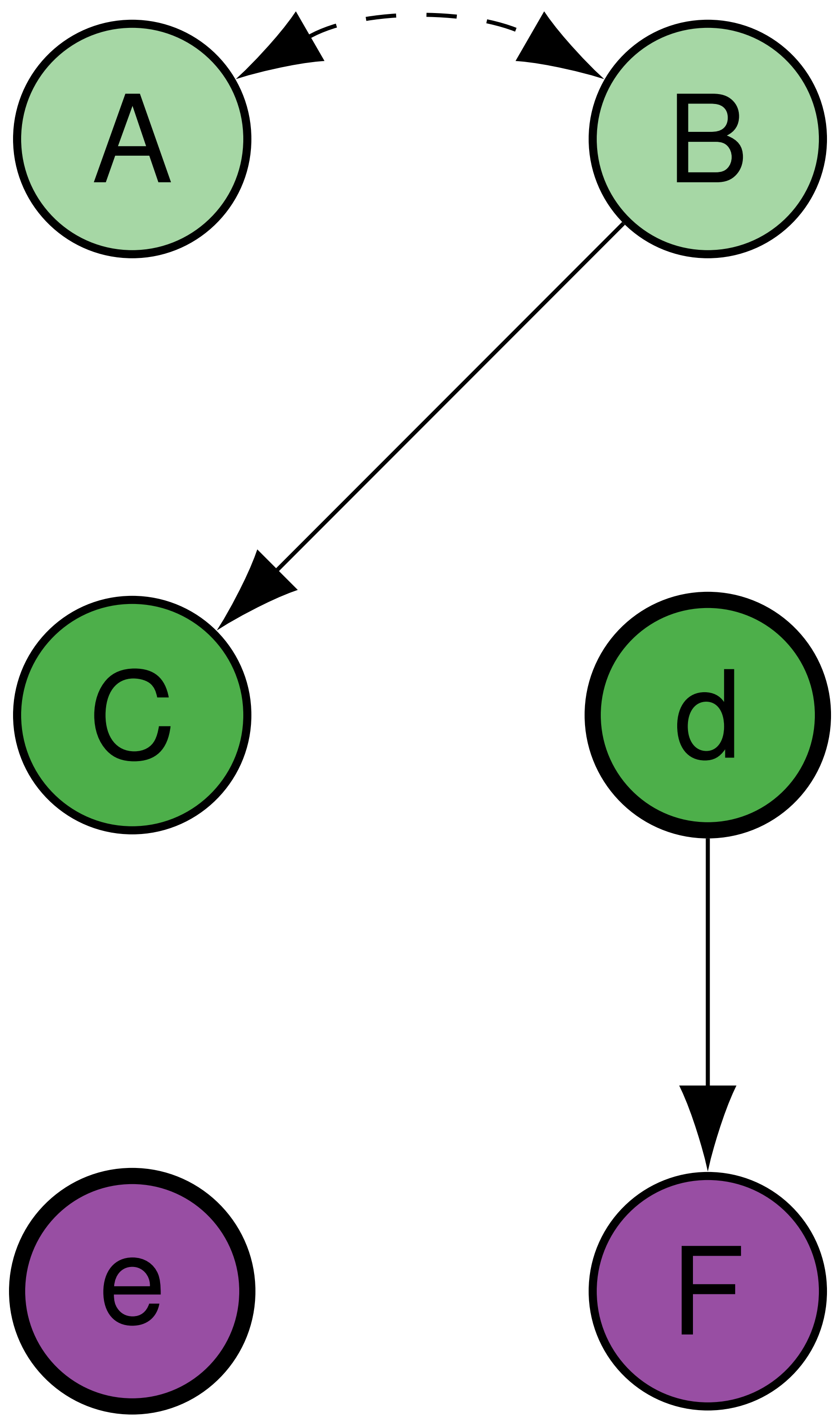

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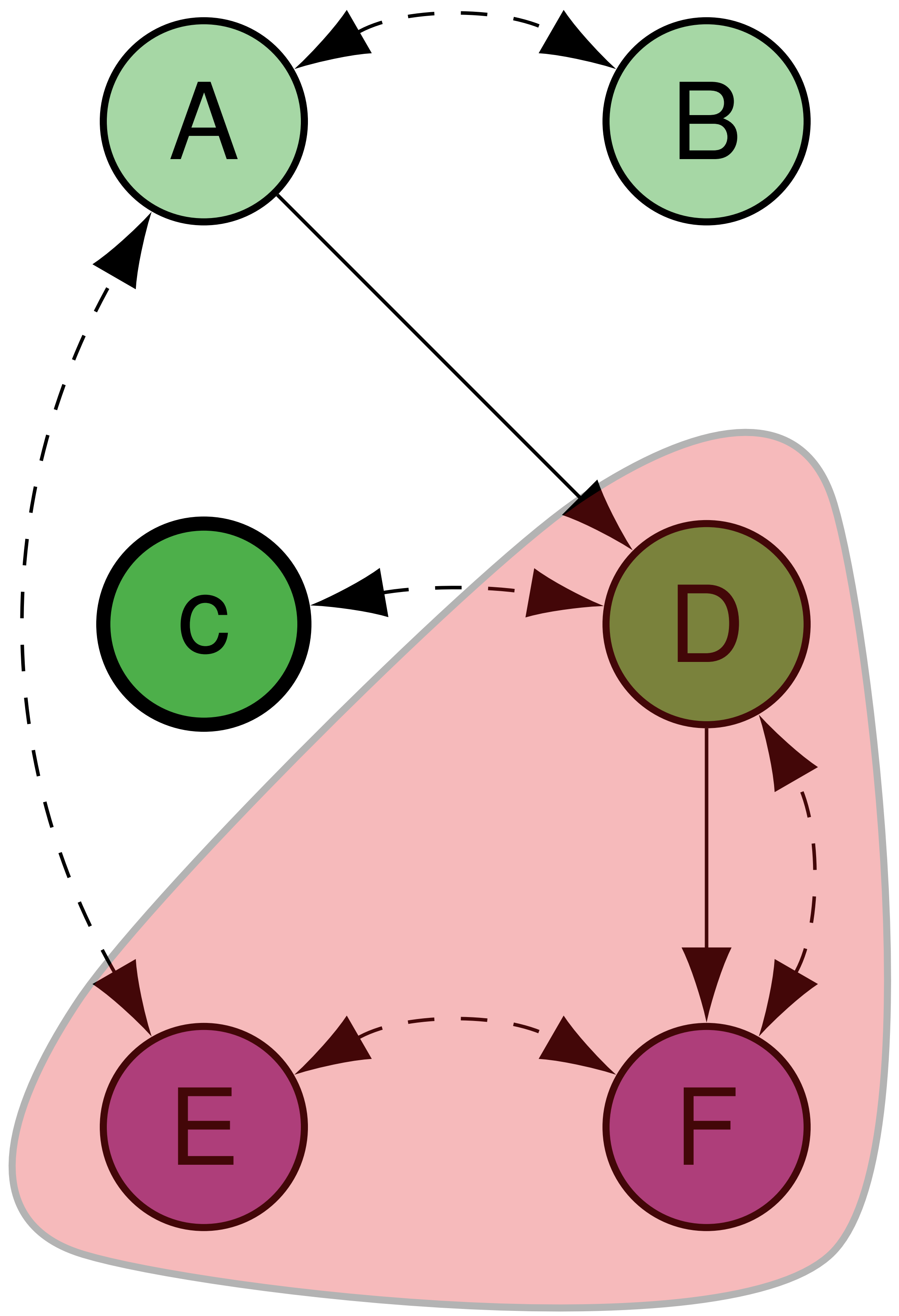
$P_{●}(●)$ versus $P_{\mathbf{Z}_i} \in \mathbb{P}$ (phase-2)

There are 3 situations in identifying a factor $P_{●}(●)$ with a distribution $P_{\mathbf{Z}_i} \in \mathbb{P}$.

- ✓ (the **good**) identified,
  e.g., P_D, $P_{c,d}(e,f) = P_d(e,f|c)$
- ✗ (the **ugly**) $\mathbf{Z}_i$ on ●,
  e.g., $P_{E,D}$
- ✗ (the **bad**) ∃**hedge** [SP'06],
  e.g., P_C



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Failure $\Rightarrow \exists$ Thicket

- A **thicket** is the superimposition of **hedges** (the bad structure).
- (if every experiment intersects with ●, confounded ●s form a 'degenerate thicket.')

Hedge: a fence or boundary formed by closely growing bushes or shrubs. **Thicket**: a dense group of bushes or trees.

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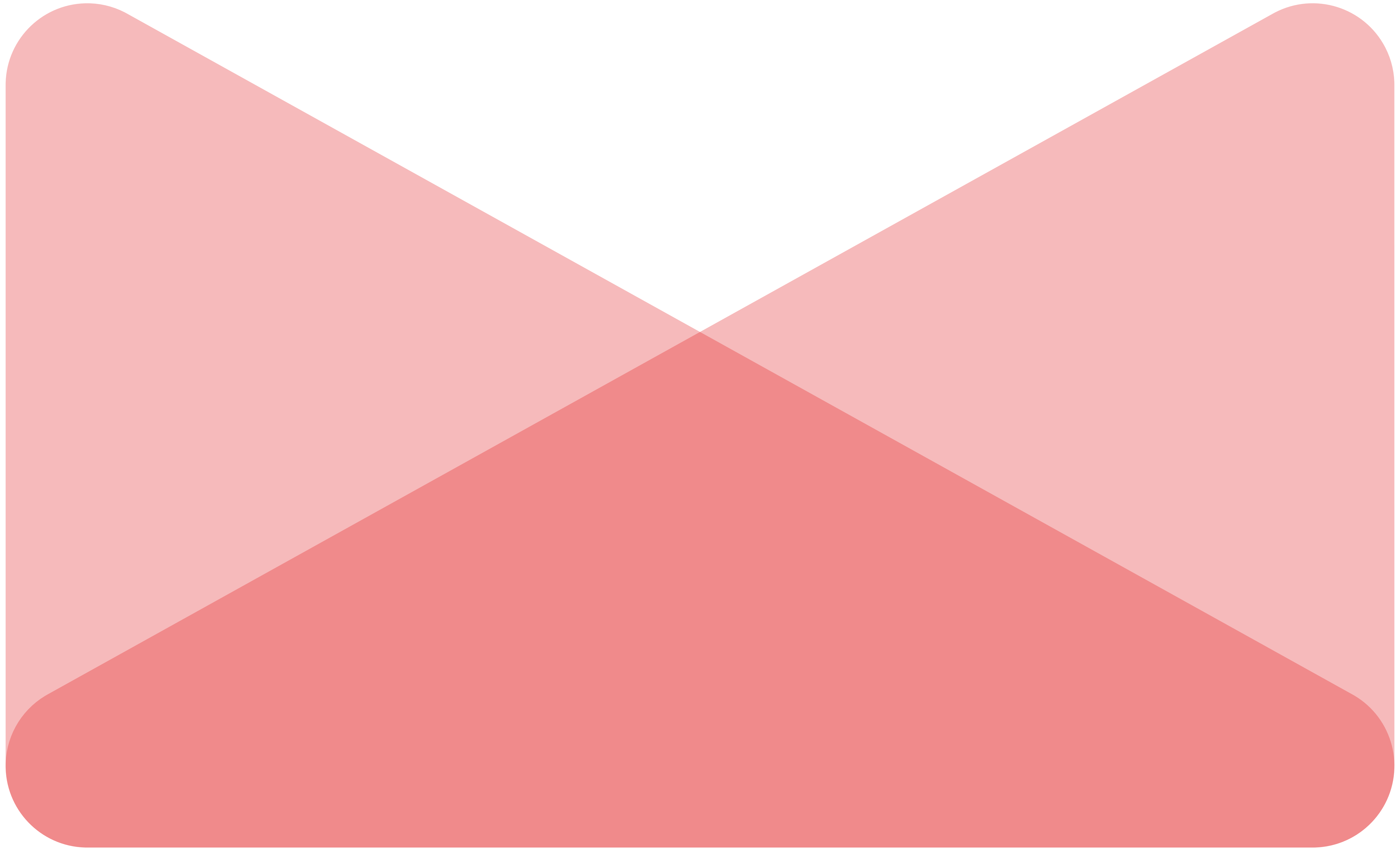

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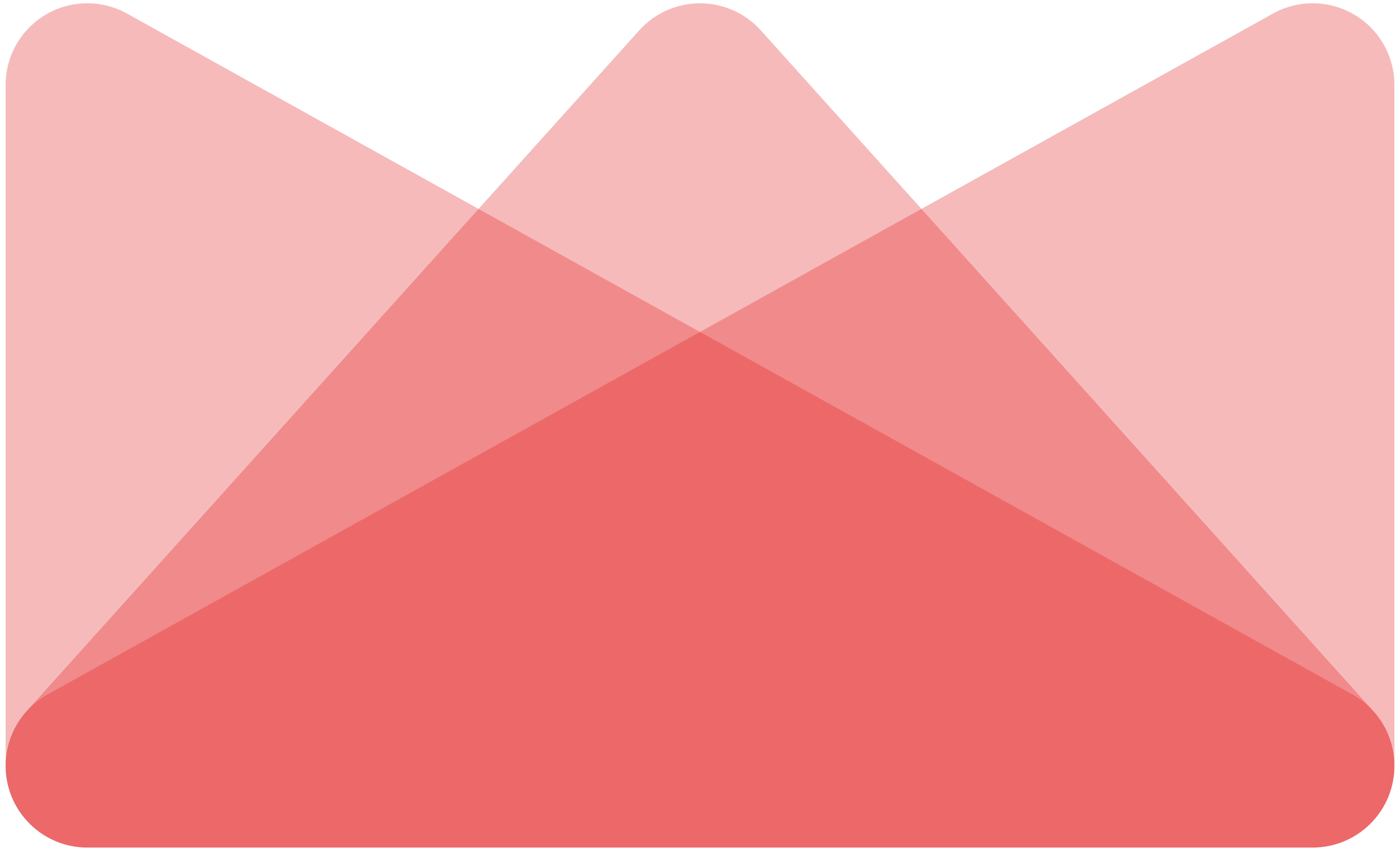

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Thicket

Definition

Let $\mathbf{R}$ be a non-empty set of variables and $\mathbb{Z}$ be a collection of sets of variables in $\mathcal{G}$. A thicket $\mathcal{J} \subseteq \mathcal{G}$ is an $\mathbf{R}$-rooted c-component consisting of a minimal c-component over $\mathbf{R}$ and hedges

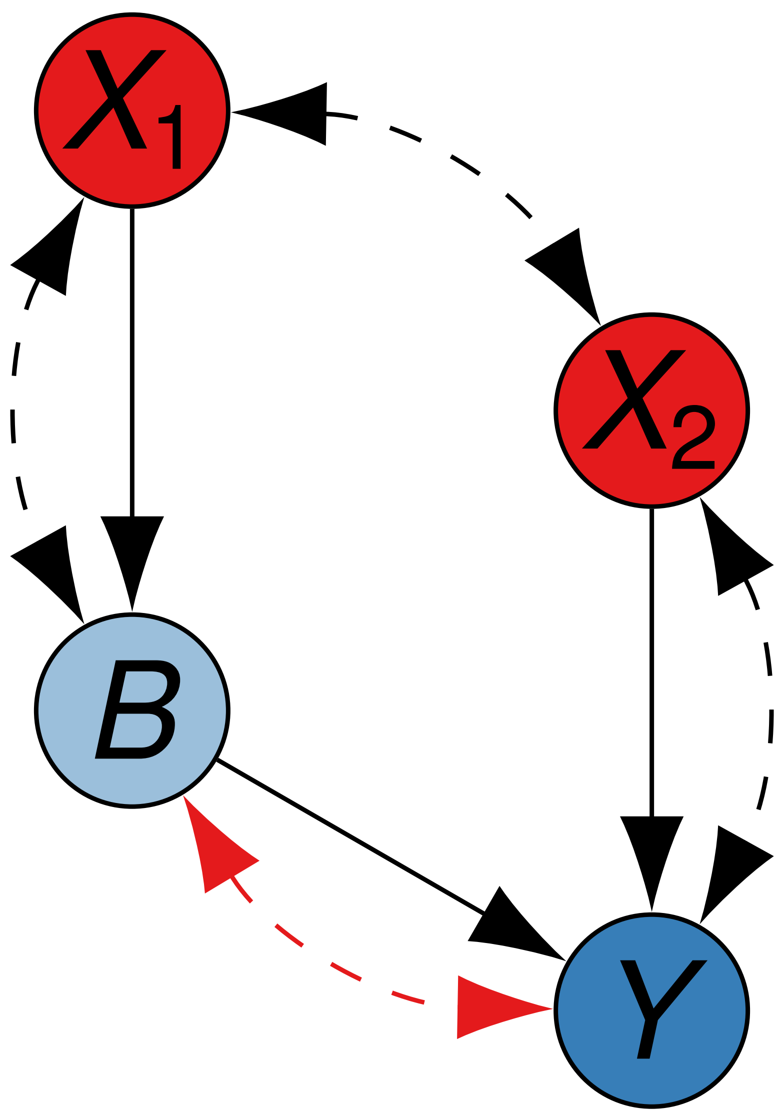
$$\mathbb{F}_{\mathcal{J}} = \{\langle \mathcal{F}_{\mathbf{Z}}, \mathcal{J}[\mathbf{R}] \rangle \mid \mathcal{F}_{\mathbf{Z}} \subseteq \mathcal{G} \setminus \mathbf{Z}, \mathbf{Z} \cap \mathbf{R} = \emptyset\}_{\mathbf{Z} \in \mathbb{Z}}.$$

Let $\mathbf{X}$, $\mathbf{Y}$ be disjoint sets of variables in $\mathcal{G}$. A thicket $\mathcal{J}$ is said to be formed for $P_{\mathbf{x}}(\mathbf{y})$ in $\mathcal{G}$ with respect to $\mathbb{Z}$ if $\mathbf{R} \subseteq An(\mathbf{Y})_{\mathcal{G}_{\underline{\mathbf{X}}}}$ and every hedgelet of each hedge $\langle \mathcal{F}_{\mathbf{Z}}, \mathcal{J}[\mathbf{R}] \rangle$ intersects with $\mathbf{X}$.

Thicket: Drug-Drug Interactions

Given $\mathbb{Z} = \{\{X_1\}, \{X_2\}\}$ with $Q = P(y|do(x_1, x_2))$ (* $P_{●}(●) = P_{●}(●)$)

- The query is not g-identifiable, see $B \leftarrow - \rightarrow Y$



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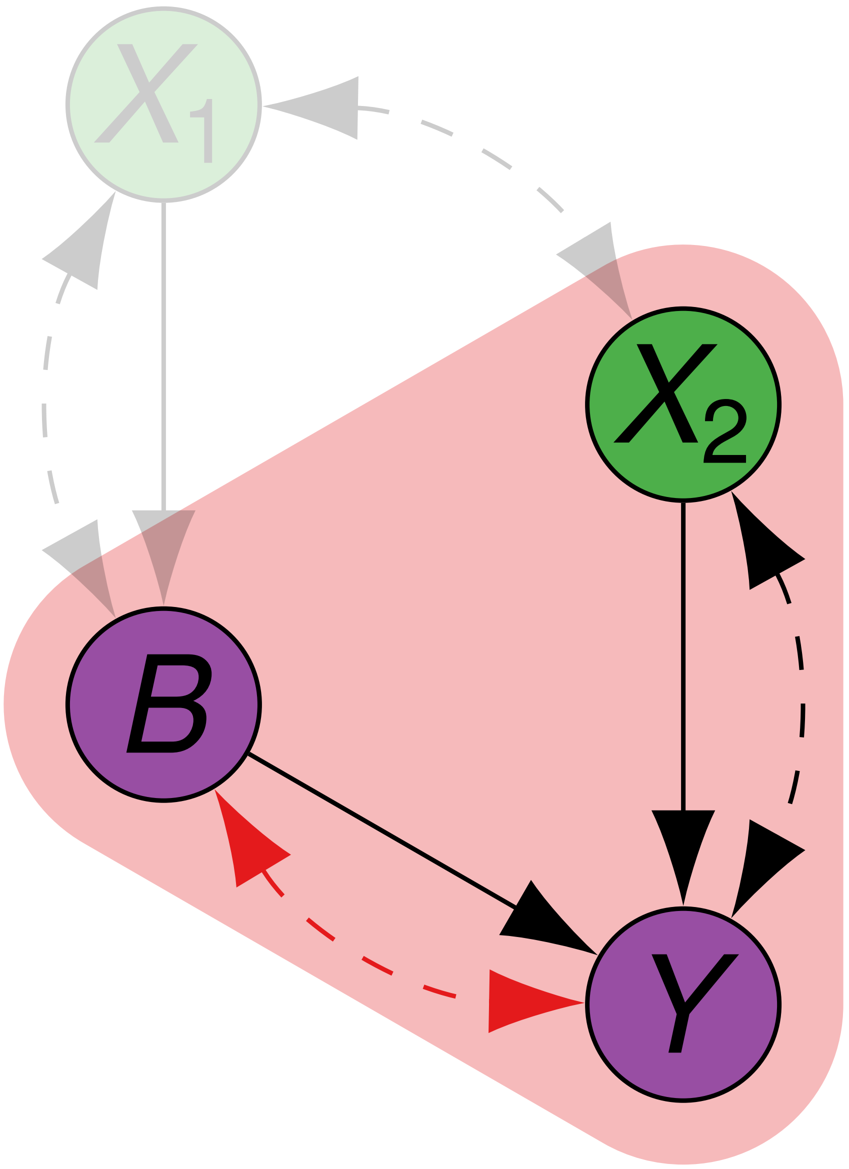
Given $\mathbb{Z} = \{\{X_1\}, \{X_2\}\}$ with $Q = P(y|do(x_1, x_2))$ (* $P_{\bullet}(\bullet) = P_{\bullet}(\bullet)$)

- There is **a hedge**, which is disjoint with $\{X_1\} \in \mathbb{Z}$, and also intersects with **X**.



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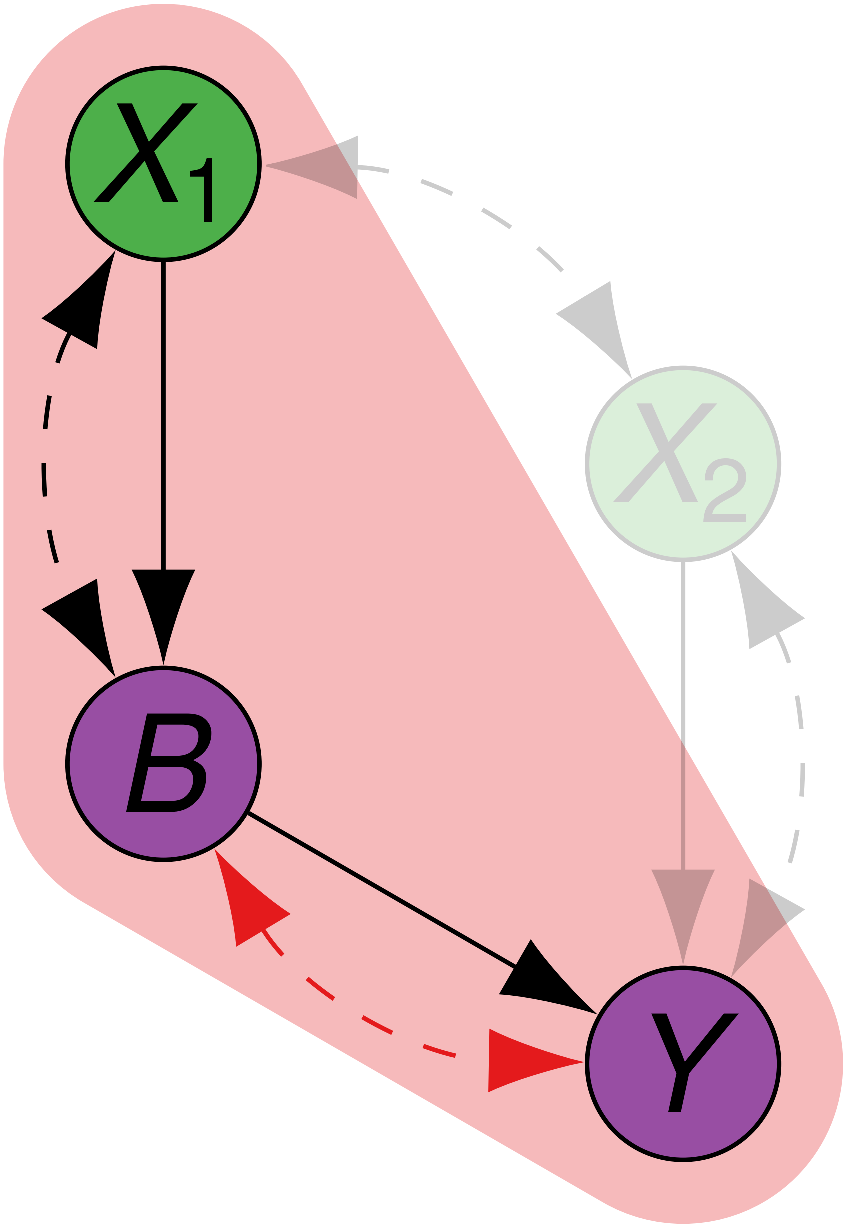
Given $\mathbb{Z} = \{\{X_1\}, \{X_2\}\}$ with $Q = P(y|do(x_1, x_2))$ (* $P_{\color{red}\bullet}(\color{blue}\bullet) = P_{\color{green}\bullet}(\color{purple}\bullet)$)

- There is **another hedge**, which is disjoint with $\{X_2\} \in \mathbb{Z}$ and, again, intersects with **X**.



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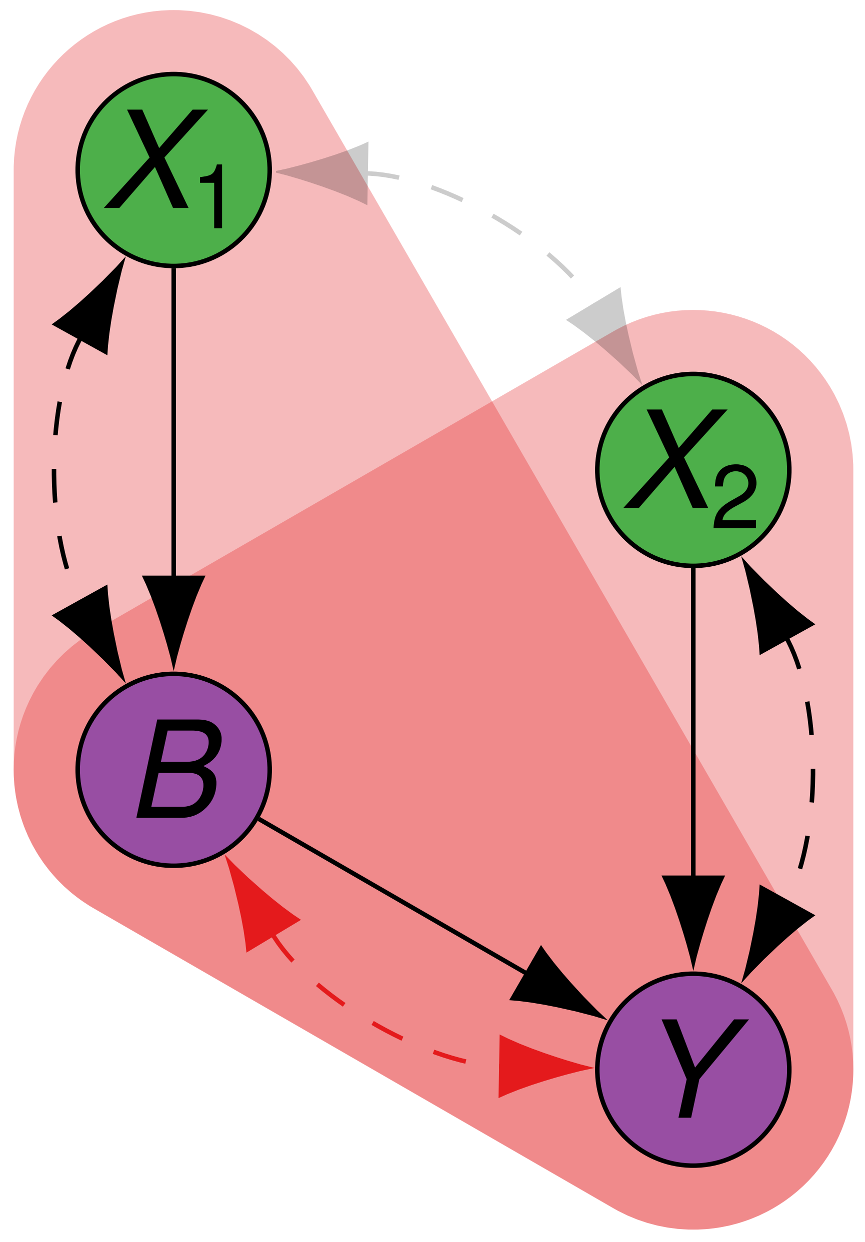

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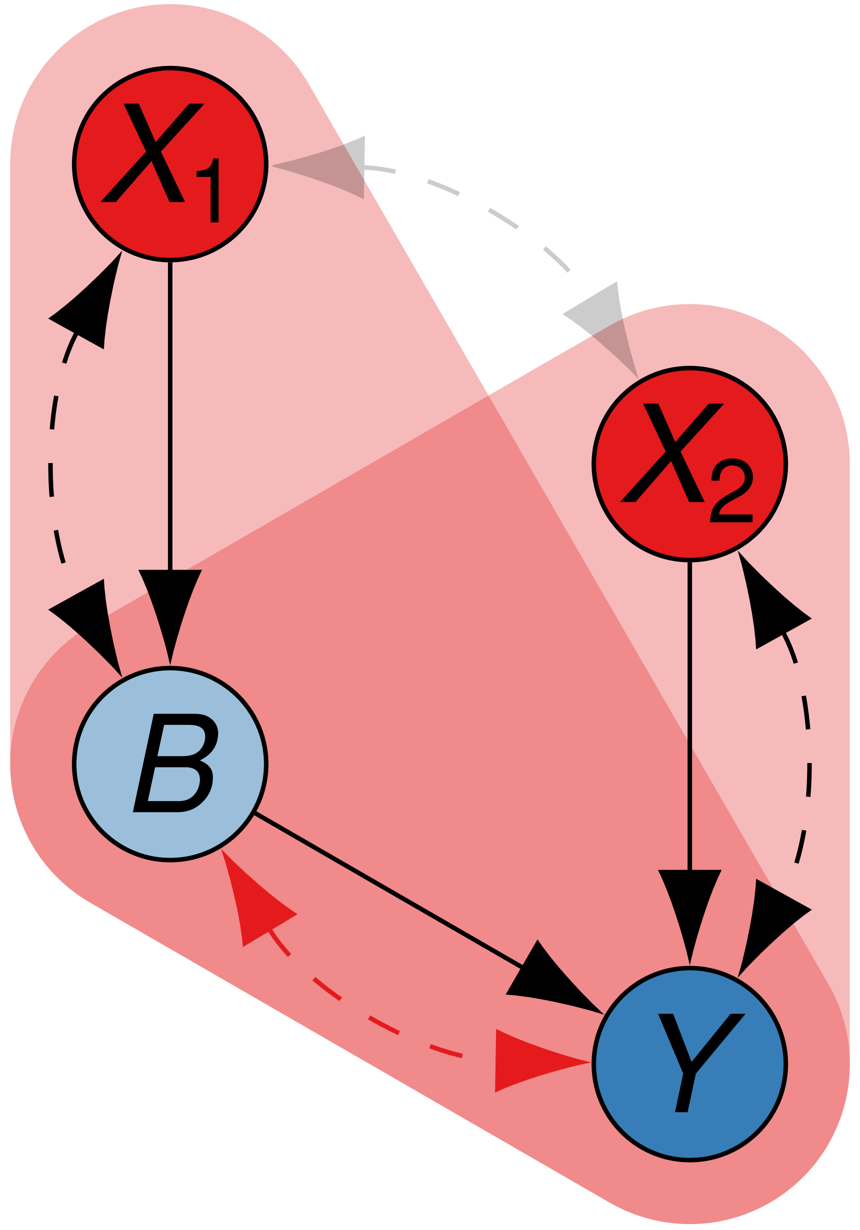

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Non-g-identifiability

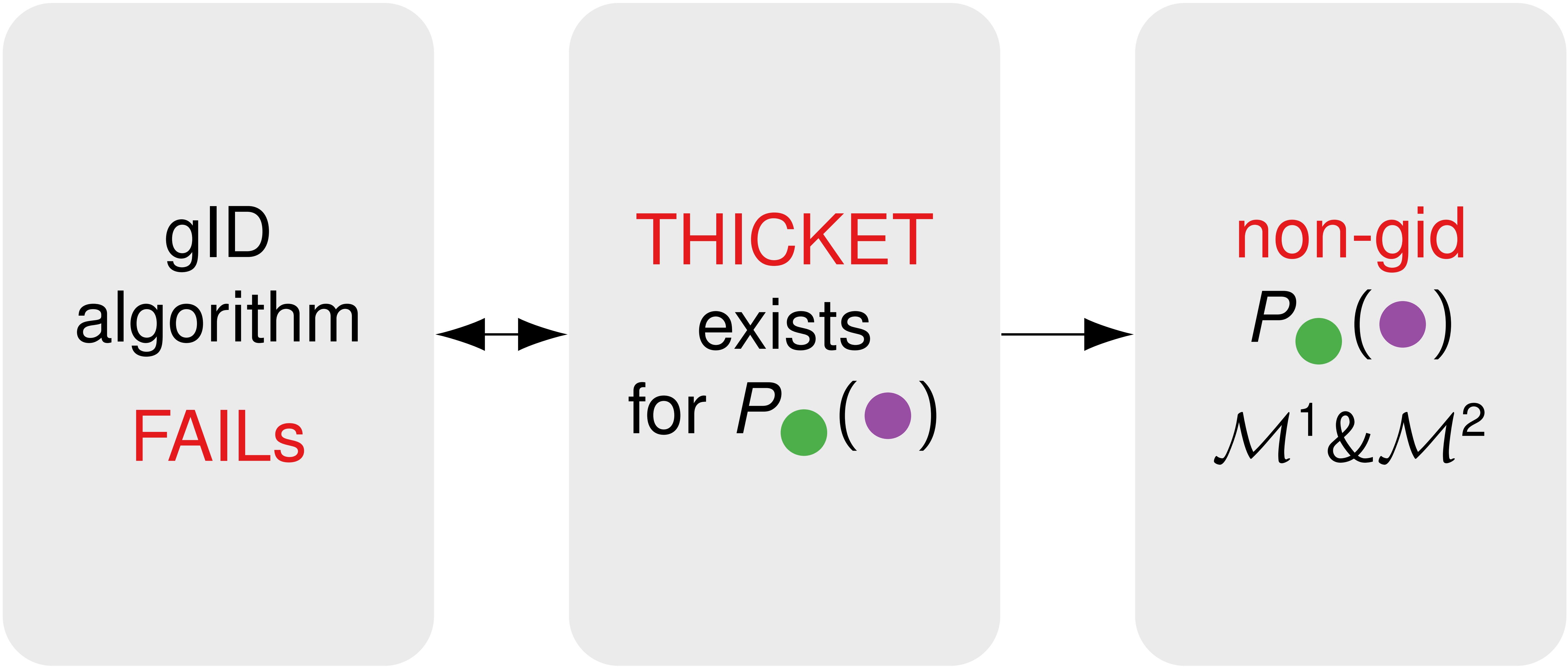
gID
algorithm

FAILs

Non-g-identifiability

gID algorithm **FAILs** $\longleftrightarrow$ **THICKET** exists for $P_{\bullet}(\bullet)$

Non-g-identifiability



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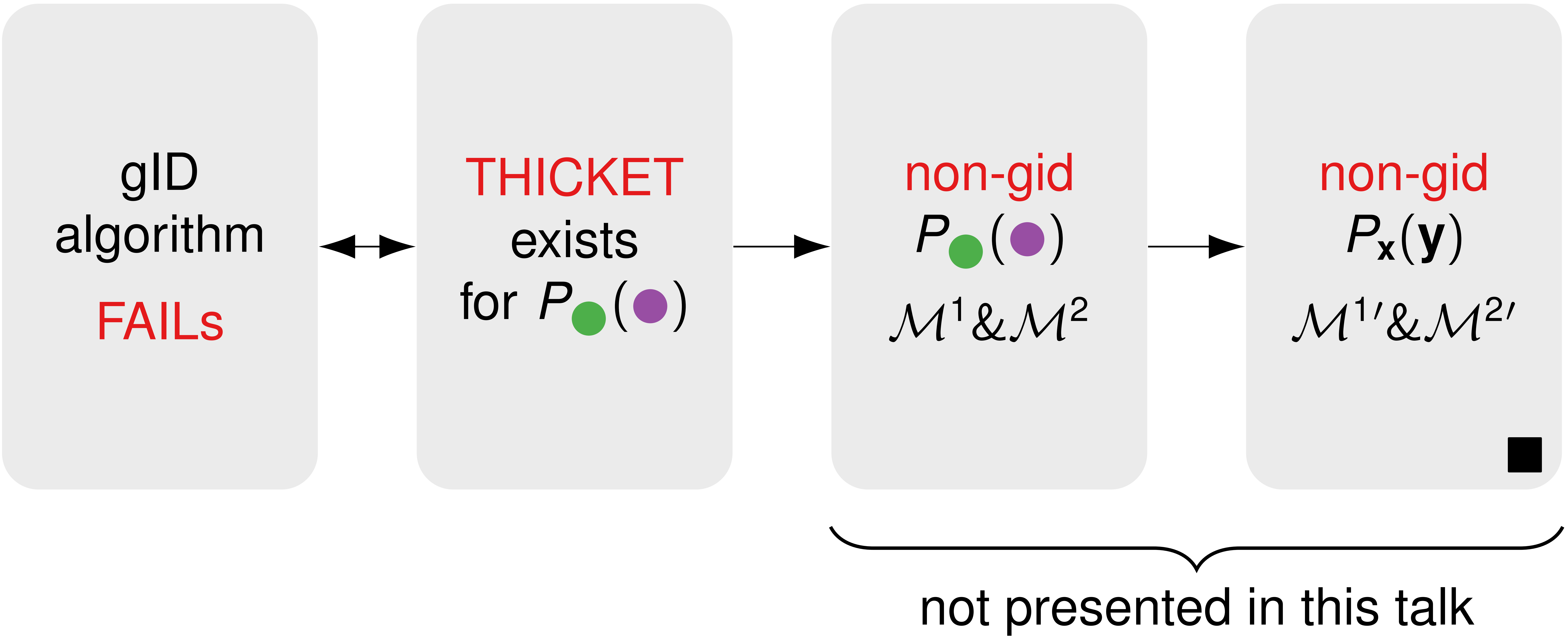

Conclusions

- We studied general-identifiability — causal effect indentifiability given a causal graph and an **arbitrary combination of observational and experimental distributions**.
- ✓ a **necessary and sufficient** graphical **condition** (∃thicket?).
  ✓ a **sound and complete algorithm**
- Research Directions: finite-sample efficient formula, studying bounds for the causal effect when not-g-identifiable, incorporating functional assumptions, without a causal graph or partially-specified graphs.
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